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FORMULAS FOR THE WEIGHTS OF BRIDGES.

BY PROFESSOR A. J. DUBOIS, JUN. AM. SOC. C. E.

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WITH DISCUSSION.

In the Society Transactions for February, 1886, Vol. XV, page 85, Mr. George H. Pegram, M. Am. Soc. C. E., presented a set of empirical formulas which excited considerable discussion and interest. The object of the present paper is to present a series of rational formulas, which accommodate any specifications and design, and which not only give accurately the weight of iron for any given style, but also are of value in deciding the best depth and length of panel in any given case. We shall first give these formulas and illustrate their use and application, and afterwards give their demonstration, so that their basis and rational form may be clearly recognized. If the method and formulas here given are what the writer at present is disposed to think them, they merit discussion, as they seem to give better results as to weight, over a larger range, than any heretofore proposed; and if thus accurate, owing to their rational form, they ought to be reliable guides in questions of practice. No other formulas known to the writer admit of such close determination of weight, give the best depth and panel length for any case, and between different styles of truss, best dimensions being taken in each case, enable one to choose the best.

Floor System.

We shall first consider the floor by itself. The chief defect hitherto, in all attempts to deduce formulas for weight, has been in not keeping the floor system separate. The floor we consider as composed of the rails, ties, planking, etc. (or in the case of highway bridges, of the roadway, etc.); and of the stringers and cross-girders. The weight of the rails, ties, planking, etc., is tolerably constant for all railroad bridges, while for highway bridges the planking, roadway, etc., must be estimated for the case in hand.

For railroad bridges we take the weight of rails, ties, planking, etc., at 400 pounds per lineal foot for single track. We take this as not far from the average simply for illustration, so that we may fully explain our method of procedure and the application of our formulas.

This portion of the floor being then settled upon for any case, we next consider the stringers and cross-girders. If the stringers are of wood they are easily estimated and need not be considered here. If, as is most always the case, we have iron plate-girders of uniform depth, we can at once estimate the weight by the following formula:

$$\left.\begin{array}{l}\text{Weight of a plate-girder}\\ \text{in pounds.}\end{array}\right\} = \frac{12\ W l^2 + 2\ R\ l\ d^2}{1.2\ R\ d - 12\ l^2} \qquad (1)$$

where l = span in feet; d = depth in inches; R = the average flange stress in pounds per square inch; W = the total external load in pounds, including allowance for impact.

This formula gives very close results, and was first given by the writer in the February number of the Transactions referred to. Mr. Pegram there tested it by actual weights, with the following results for $R = 9\,000$ pounds.

Span in Feet.	Depth in Inches.	W.	Formula Weight.	Actual Weight.
20	24	68 700	2 110	2 645
30	36	96 000	4 594	4 871
40	42	110 000	7 772	7 818
40	48	110 000	7 554	7 824
50	54	130 000	11 786	11 835
50	60	130 000	11 550	11 882
60	60	144 000	16 714	16 432
60	66	144 000	16 307	16 614
70	72	156 870	21 962	21 117
80	72	172 000	29 446	28 535
80	84	172 000	28 097	28 114

These results, as Mr. Pegram remarks, are "all that could be asked." Such differences as there are can be as well attributed to a change in the value of R as to any defect in the formula.

From formula (1) we have directly the best or economic depth.

$$\text{Economic depth in inches} = \frac{10\,l^2}{R} + \sqrt{\frac{6\,W\,l}{R} + \left(\frac{10\,l^2}{R}\right)^2} \qquad (2)$$

From formula (2) we can find the best depth in inches, d, of a plate girder, when the total external load, W, in pounds, length in feet, l, and working stress in pounds per square inch, R, are known. The weight, as shown by (1), will vary but little for values of d, differing not much from the best depth.

These two formulas are not difficult or laborious of application, but such labor as may be involved in their application may be greatly lessened by tables constructed from them in accordance with standard specifications.

We shall, for the sake of complete illustration, give such tables. The live-load system we assume for this purpose, consists of two typical locomotives with tenders, followed by train, as given in "Strains in Framed Structures," second edition, Wiley & Sons. We have there given tables by which moments and equivalent uniform loads can be at once and accurately determined for the actual weights and distances. We give this system below. It will be seen that it is somewhat in excess of Mr. Pegram's typical consolidated Class T, which he states "is now coming into use, and will probably be the loading for some time to come."

LIVE-LOAD SYSTEM ASSUMED FOR ILLUSTRATION.

LOCOMOTIVE.		LOCOMOTIVE.		CAR.	
15 000		15 000		12 000	
	8 feet.		8 feet.		5 feet.
25 000		25 000		12 000	
	5 "		5 "		10 "
25 000		25 000		12 000	
	5 "		5 "		5 "
25 000		25 000		12 000	
	5 "		5 "		5 "
25 000		25 000			
	7 "		7 "		
TENDER.		TENDER.		CAR.	
15 000		15 000		12 000	
	5 "		5 "		5 "
15 000		15 000		12 000	
	5 "		5 "		10 "
15 000		15 000		12 000	
	5 "		5 "		5 "
15 000		15 000		12 000	
	0 "		4 "		5 "
					etc.

For impact we take 30 per cent. increase of external load for all spans below 25 feet and $40 - \frac{2}{3} l$ per cent. increase for all spans above 25 feet (l = span in feet). We take rails, ties, etc., at 200 pounds per foot per stringer, or 400 pounds per foot per track, and R at 8 000 pounds per square inch. If these data are considered satisfactory, those who wish to use our method may in general use the following tables, and will seldom need to calculate at all.

TABLE No. 1.

Iron-plate Stringers of Uniform Depth.

Panel Length in Feet.	Equivalent Uniform Live Load per Stringer in Pounds.	Total External Load W, including Allowance for Impact and Flooring at 200 pounds per foot.	Economic Depth in Inches.	Weight in Pounds.
10	25 000	35 100	16.4	545
11	27 272	38 314	18	657
12	33 333	46 453	20.6	828
13	36 538	50 879	22.5	974
14	39 286	54 711	24.2	1 130
15	41 666	58 066	25.8	1 292
16	43 750	61 035	27.4	1 460
17	45 588	63 684	28.8	1 634
18	47 222	66 068	30.3	1 816
19	48 684	68 230	31.6	2 000
20	50 000	70 200	33	2 197
21	52 380	73 554	34.6	2 421
22	54 545	76 628	36.2	2 652
23	56 521	79 457	37.7	2 900
24	58 333	82 073	39.2	3 133
25	60 000	84 500	40.7	3 382
26	61 537	86 491	42	3 633
27	63 518	89 042	43.4	3 904
28	65 355	91 390	44 8	4 180
29	67.068	93 562	46.2	4 463
30	68 832	95 784	47.5	4 756

From this table we can take at once the weight of stringer for any panel length, in accordance with the live-load system assumed, and the allowance for impact and unit stress. In like manner, and based upon the same data, we may calculate from our formulas (1) and (2) the best depth and weight of cross-girders, assuming that each cross-girder carries two stringers as given by Table No. 1.

We give here two tables for cross-girders. The first gives the equivalent live load and total external load W, and the second, the corresponding weights for single and double track, with best depths. Again we call attention to the fact that considerable deviations from these depths will not much affect the weights.

TABLE No. 2A.

Iron-plate Cross-Girders of Uniform Depth.

Live load and total external load, W, for single track. For double track, take double values. Rails, ties, etc., 400 pounds per foot.

Panel length in Feet.	Equivalent Uniform Live Load.	Total External load W, including Live Load, weight of two Stringers and Allowance for Impact.	Panel length in Feet.	Equivalent Uniform Live Load.	Total External Load, W, including Live Load, weight of two Stringers and Allowance for Impact.
10	60 000	71 620	21	87 619	131 400
11	64 545	78 340	22	90 226	135 630
12	68 332	84 220	23	93 260	140 740
13	72 690	90 790	24	96 041	145 480
14	76 428	96 575	25	98 400	150 040
15	80 666	101 725	26	100 960	154 220
16	82 500	106 370	27	103 147	158 280
17	86 000	110 590	28	105 714	162 860
18	89 055	115 560	29	108 103	167 220
19	92 578	121 140	30	110 333	171 400
20	94 750	126 290			

TABLE No. 2B.

Iron-plate Cross-Girders.

Weight and economic depth for single track, 15 feet wide, and double track, 25 feet wide. Rails, ties, etc., 400 pounds per foot. $R = 8\,000$; allowance for impact, 30 per cent.

Panel length in Feet.	Single Track, 15 feet wide.		Double Track, 25 feet wide.		Panel length in Feet.	Single Track, 15 feet wide.		Double Track, 25 feet wide.	
	Depth in Inches.	Weight in Pounds.	Depth in Inches.	Weight in Pounds.		Depth in Inches.	Weight in Pounds.	Depth in Inches.	Weight in Pounds.
10	29	1 433	52.6	4 384	21	38.7	1 937	71	5 915
11	30	1 500	55	4 582	22	39	1 967	72	6 000
12	31	1 553	57	4 748	23	40	2 000	73.4	6 060
13	32	1 612	59	4 886	24	40.7	2 036	74.6	6 160
14	33	1 662	61	5 080	25	41.4	2 068	75.8	6 260
15	34	1 706	62.5	5 212	26	42	2 096	76.8	6 345
16	35	1 744	64	5 328	27	42.4	2 124	77.8	6 427
17	35.5	1 772	65	5 400	28	43	2 154	79	6 564
18	36.3	1 811	66.5	5 551	29	43.6	2 172	80	6 664
19	37.2	1 860	68.2	5 682	30	44.2	2 209	81	6 748
20	38	1 898	69.6	5 743					

If the widths are very different from those assumed, it may be necessary to use formulas (1) and (2). In general, for highway bridges,

these formulas must be used. But for ordinary railroad practice, Tables Nos. 1 and 2 will give at once weight of stringers and cross-girders.

A good rough and ready rule for depth of cross-girder for single track is: depth of cross-girder in inches, = panel length in feet, + width of roadway in feet. For double track about 1.6 times this depth.

Our two tables, or the formulas (1) and (2) from which they are derived, thus enable us at once to estimate readily the weight of iron in the floor system.

We are now able to estimate the total weight of iron, as well as to determine the best depth for any *plate-girder bridge*.

Plate-Girder Bridges.

For this purpose we need only know the equivalent uniform live load and the weight of wind bracing.

The weight of wind bracing is given by the empirical formula

$$\text{total weight of wind bracing} = N\,(540 + 3.6\,l)$$

where N is the number of panels, and l = span in feet.

For spans under 30 feet, the equivalent uniform live load for the whole bridge may be found from Table No. 1 by taking double the values there given. Above 30 feet we have the following.

TABLE No. 3.

Equivalent Uniform Live Load for Plate-Girder Bridges.

Span in Feet.	Equivalent Uniform Live Load.	Span in Feet.	Equivalent Uniform Live Load.	Span in Feet.	Equivalent Uniform Live Load.	Span in Feet.	Equivalent Uniform Live Load.
30	137 670	43	184 580	56	217 300	69	244 750
31	140 310	44	186 940	57	218 770	70	246 770
32	143 130	45	189 300	58	222 040	71	248 530
33	152 700	46	192 860	59	224 900	72	250 470
34	154 950	47	195 190	60	227 770	73	252 380
35	157 000	48	198 420	61	229 860	74	253 940
36	158 900	49	200 340	62	231 820	75	256 070
37	163 000	50	203 500	63	233 700	76	258 470
38	170 000	51	205 270	64	235 520	77	260 760
39	172 800	52	207 970	65	237 280	78	262 770
40	176 280	53	209 790	66	239 130	79	264 490
41	179 190	54	212 480	67	240 850	80	267 120
42	181 890	55	213 960	68	242 500		

Let us now illustrate the preceding by an example:

Example.—Required the best depth and weight of iron for a single track through plate-girder bridge, 63 feet long and 15 feet wide, center

to center. Distance between floor beams, 15¾ feet. Iron-plate stringers and floor beams.

As there are four panels, the wind bracing is $4\,(540 + 3.6 \times 63) =$ 3 067 pounds, or 1 533 pounds for each girder, if there are two. Each stringer is 15¾ feet long, or, allowing for thickness of cross-girders, nearer 15 feet long, and from Table No. 1 we have for the weight of such a stringer 1 292, or, say, 1 300 pounds. As there are eight such stringers, the total weight is 10 400 pounds, or 5 200 pounds for each girder. The weight of cross-girders from Table No. 2 is about 1 700 pounds. There are five of these, and hence 4 250 pounds for each girder. The rails, ties, etc., we take at 200 pounds per foot for each girder, or 12 600 pounds. The live load from Table No. 3 gives 116 850 pounds per girder. Total, 140 433. Allowance for impact, $40 - \frac{2}{5}\,l = 14.8$ per cent., or 20 784.

Hence $W = 161\,217$ pounds. (If we take 14.8 per cent. of the equivalent live load only, we should have $W = 157\,700$ pounds.)

We have then from formula (2) for the best depth, taking $R = 8\,000$,

$d =$ about 92.5 inches $= 7.7$ feet,

and from formula (1), using this depth,

weight of one girder = about 19 400 pounds.

The total weight of iron is then $30\,383 \times 2 = 60\,766$ pounds. The static load for each girder is $30\,383 + 12\,600 = 42\,983$, and the girders can now be designed. If the stringers were wood, we should estimate their weight and proceed as above.

It will be seen from this example that the amount of calculation required is slight and quickly made. The estimate is close, and we have also determined the best depth. As to this latter, a considerable change in depth has but slight influence on the weight. Thus, for 60 inches depth, instead of 92.5, we have 21 400 pounds for the weight, instead of 19 400. We believe, however, that careful designing will show our depth to be the best. Our method can be adapted readily to any specifications and loading. It gives not only a close value for weight, but is an aid in designing. Our tables, we believe, give results in accord with good practice.

We can now pass on to

Bridge Trusses.

Let $w_1 =$ the equivalent uniform load per foot per truss due to the assumed live load system.

$w_2 =$ load per foot per truss due to the cross-girders, stringers, and rails, ties, planking, etc.

$w_3 =$ load per foot per truss due to the wind bracing.

These three, $w_1 + w_2 + w_3$, give the total external load per foot per truss.

For our assumed load system, we have the following values of w_1 and w_2:

TABLE No. 4.

EQUIVALENT UNIFORM LOAD, w_1, PER FOOT PER TRUSS, ON THE BASIS OF ASSUMED LOAD SYSTEM FOR SINGLE TRACK. FOR DOUBLE TRACK, TAKE DOUBLE VALUES.

Span =	60	65	70	80	90	100	110	120	130
w_1 =	1 845	1 782	1 723	1 636	1 623	1 625	1 643	1 637	1 625
Span =	140	150	160	170	180	190	200	210	220
w_1 =	1 610	1 588	1 560	1 538	1 511	1 486	1 460	1 435	1 408
Span =	230	240	250	260	270	280	290	300	310
w_1 =	1 386	1 370	1 348	1 329	1 310	1 292	1 275	1 260	1 245
Span =	320	330	340	350	360	370	380	390	400
w_1 =	1 230	1 215	1 200	1 185	1 170	1 155	1 140	1 125	1 110

These equivalent uniform loads can be easily checked from our tables in "Strains in Framed Structures," second edition. The table, it will be noticed, gives w_1 for one truss, on the assumption of two trusses to the bridge.

Taking the weight of rails, ties, etc., at 200 pounds per truss, and making use of Tables 1 and 2 for the weight of stringers and cross-girders, we have the following values for w_2. (See Table No. 5.)

Finally, for the load in pounds per foot per truss due to wind bracing, w_3, we have the following empirical formulas:

Depth below 12.5 feet or pony trusses,

$$w_3 = \frac{N(270 + 1.8\,l)}{l}$$

Depth between 12.5 and 24 feet, upper and lower horizontal bracing,

$$w_3 = \frac{N(336 + 3.2\,l)}{l}$$

Depth above 24 feet, upper and lower horizontal and vertical sway bracing,

$$w_3 = \frac{N(7.5\,l - 180)}{l}$$

where N = number of panels and l = span in feet. These formulas

TABLE No. 5.

LOAD w_2 PER FOOT PER TRUSS FOR SINGLE AND DOUBLE TRACKS.

Panel Length in Feet.	Single Track.				Double Track.			
	Weight of One half Cross-girder.	Weight of One Stringer.	Weight of One-half Flooring, etc.	w_2	Weight of One-half Cross-girder.	Weight of Two Stringers.	Weight of One Floor, etc.	w_2
5	507	188	1 000	339	1 555	376	2 000	786
6	582	248	1 200	338	1 783	496	2 400	780
7	631	315	1 400	335	1 935	630	2 800	766
8	679	386	1 600	333	1 992	772	3 200	745
9	695	463	1 800	327	2 135	926	3 600	740
10	716	545	2 000	326	2 192	1 090	4 000	728
11	750	657	2 200	328	2 291	1 314	4 400	727
12	776	825	2 400	333	2 374	1 650	4 800	735
13	806	974	2 600	337	2 443	1 948	5 200	738
14	831	1 130	2 800	340	2 540	2 260	5 600	743
15	853	1 292	3 000	343	2 606	2 584	6 000	746
16	872	1 460	3 200	345	2 664	2 920	6 400	748
17	886	1 634	3 400	348	2 700	3 268	6 800	751
18	906	1 816	3 600	351	2 775	3 632	7 200	755
19	930	2 003	3 800	354	2 841	4 006	7 600	760
20	949	2 197	4 000	357	2 871	4 394	8 000	763
21	968	2 420	4 200	361	2 957	4 840	8 400	771
22	983	2 652	4 400	365	3 004	5 304	8 800	777
23	1 001	2 900	4 600	369	3 030	5 800	9 200	783
24	1 018	3 133	4 800	373	3 080	6 266	9 600	789
25	1 034	3 382	5 000	376	3 128	6 764	10 000	795
26	1 048	3 633	5 200	380	3 172	7 266	10 400	801
27	1 062	3 904	5 400	384	3 213	7 808	10 800	808
28	1 077	4 180	5 600	387	3 282	8 360	11 200	816
29	1 086	4 463	5 800	391	3 332	8 926	11 600	822
30	1 104	4 756	6 000	395	3 373	9 512	12 000	829

are for width of 15 feet. If width is greater, multiply by $\frac{\text{width}}{15}$

The last formula holds for deck bridges also.

We can thus find, for any case, the value of w_1, w_2 and w_3 for railroad bridges. For highway bridges w_3 is the same, but w_2 and w_1 will be different according to construction and case. No trouble in any given case will be found in finding w_1 and w_2. Now the total external load, $w_1 + w_2 + w_3$, being thus known, our formula for the weight per foot of one truss, w_4, not including bed plates, rollers and end shoes, is

$$w_4 = \frac{w_1 + w_2 + w_3}{\dfrac{3.6\ \mu\ d}{A + \dfrac{\mu\ (45\ p^2 + 202\ d^2)}{(w_1 + w_2 + w_3)\ p}} - 1} \qquad (3)$$

In this formula, w_1, w_2 and w_3 have already been defined and their

values given; d = depth of truss in feet; p = panel length in feet; μ = the numerator of the strut formula used. Thus the strut formula specified by Mr. Pegram, and generally used, is for iron, $\frac{8\,000}{1+\frac{(\text{length})^2}{a\,r^2}}$, where length is in inches, r is the least radius of gyration in inches, and a is a constant depending upon the end conditions. In such case μ in formula (3) stands for 8 000. In general μ stands for the numerator of the strut formula used in the design. The only remaining quantity in our formula is A. This is a theoretic quantity depending on the style of truss, number of panels, depth, and panel length, as follows:

For single intersection Pratt:

$$A = p^2\left(2\,N^2+3\,N-2\right)+3\,d^2\left(2\,N-4+\frac{11}{N}\right)$$

For double intersection Whipple:

$$A = 2\,p^2\left(N^2+3\,N-10+\frac{12}{N}\right)+3\,d^2\left(N-2+\frac{16}{N}\right)$$

For Warren girder:

$$A = p^2\,(2\,N^2+1.5\,N-2)+6\,N\,d^2$$

and so on. For every style of truss, A can easily be worked out. We present for discussion only these three types.

Our formula (3) is rational in form. The strut formula is included in it, *so that account is taken of the material required for stiffening.* The quantity 3.6 is a result of the theoretic discussion. The only empiric quantities are 45 and 202. These may vary somewhat with individual design, and have been determined by comparison with two cases where the weight and dimensions were known. The method by which the formula has been deduced, we may say here, is similar to that adopted by Charles Bender, C. E., in "Principles of Economy in the Design of Metallic Bridges." Wiley & Sons, 1885. Our value of $\frac{A}{d}$ is his "strain length," but in formula (3) we have, as we have said, *introduced the strut formula,* and thus have a variable unit stress. The fault of all formulas for weight has been the adoption of a constant unit stress for all members. We shall notice this point more at length later on. We confine ourselves here to explaining and illustrating the use of the formula. For ready application, we give in the following table the value of A for different numbers of panels, N.

TABLE No. 6.

VALUES OF A FOR DIFFERENT TRUSSES.

N.	A. SINGLE INTERSECTION.	A. DOUBLE INTERSECTION.	A. WARREN GIRDER.
2	$12\,p2 + 1\,.5\,d2$	$12\,p2 + 24\,d2$	$9\,p2 + 12\,d2$
3	$25\,p2 + 1\,d2$	$24\,p2 + 19\,d2$	$20.5\,p2 + 18\,d2$
4	$42\,p2 + 2\,.25\,d2$	$42\,p2 + 18\,d2$	$36\,p2 + 24\,d2$
5	$63\,p2 + 2\,.6\,d2$	$64.8\,p2 + 18.6\,d2$	$55.5\,p2 + 30\,d2$
6	$88\,p2 + 3\,.5\,d2$	$92\,p2 + 20\,d2$	$79\,p2 + 36\,d2$
7	$117\,p2 + 3\,.714\,d2$	$123.428\,p2 + 21.857\,d2$	$106.5\,p2 + 42\,d2$
8	$150\,p2 + 4\,.125\,d2$	$159\,p2 + 24\,d2$	$138\,p2 + 48\,d2$
9	$187\,p2 + 4\,.666\,d2$	$198.666\,p2 + 26.333\,d2$	$173.5\,p2 + 54\,d2$
10	$228\,p2 + 5\,.3\,d2$	$262.4\,p2 + 28.8\,d2$	$213\,p2 + 60\,d2$
11	$273\,p2 + 5\,d2$	$290.1818\,p2 + 31.3636\,d2$	$266.5\,p2 + 66\,d2$
12	$322\,p2 + 6\,.666\,d2$	$342\,p2 + 34\,d2$	$304\,p2 + 72\,d2$
13	$375\,p2 + 6\,.538\,d2$	$397.846\,p2 + 36.69\,d2$	$355.5\,p2 + 78\,d2$
14	$432\,p2 + 7\,.357\,d2$	$457.714\,p2 + 39.4285\,d2$	$411\,p2 + 84\,d2$
15	$493\,p2 + 8\,.2\,d2$	$521.6\,p2 + 42.2\,d2$	$470.5\,p2 + 90\,d2$
16	$558\,p2 + 8\,.0626\,d2$	$589.5\,p2 + 45\,d2$	$534\,p2 + 96\,d2$
17	$627\,p2 + 9\,.941\,d2$	$661.412\,p2 + 47.8235\,d2$	$601.5\,p2 + 102\,d2$
18	$700\,p2 + 9\,.833\,d2$	$737.333\,p2 + 50.666\,d2$	$673\,p2 + 108\,d2$
19	$777\,p2 + 10\,.737\,d2$	$817.263\,p2 + 53.5263\,d2$	$748.5\,p2 + 114\,d2$
20	$858\,p2 + 10\,.65\,d2$	$901.4\,p2 + 56.4\,d2$	$828\,p2 + 120\,d2$

We can now readily apply formula (3).

The following tabulation gives the results of our formula as applied to the cases given by Mr. Pegram in the February, 1886, number of the Transactions, taking $\mu = 8\,000$:

Span in Feet.	Depth in Feet.	Panel Length in Feet.	$w_1 + w_2 + w_3$ as given.	Formula Weight of One Truss.	Actual Weight of One Truss.	Difference.	Per Cent. of Difference.
104 S. I.	24	17⅓	1 820	23 525	23 658	— 133	$-0\frac{56}{100}$
			2 000	24 726	24 458	+ 268	$+1\frac{8}{100}$
			2 090	25 330	25 074	+ 256	$+1\frac{7}{100}$
150 D. I.	25	16⅔	1 675	46 000	46 032	— 32	$-0\frac{7}{100}$
			1 844	48 688	49 628	— 940	$-1\frac{89}{100}$
			1 950	50 388	52 627	— 2 239	$-4\frac{25}{100}$
201½ D. I.	28	16′ 9½″	1 565	87 499	85 131	+ 2 368	$+2\frac{78}{100}$
			1 776	94 872	93 626	+ 1 246	$+1\frac{33}{100}$
			1 946	100 879	100 283	+ 596	$+0\frac{59}{100}$
320 D. I.	34	20	1 605	286 445	273 671	+ 12 774	$+4\frac{66}{100}$
			1 798	312 694	299 718	+ 12 976	$+4\frac{33}{100}$
			2 088	351 475	302 206	+ 49 269?	?
255½ D. I.	29	18½	1 943	186 392	185 558	+ 834	$+0\frac{4}{100}$
	38	18½	1 943	169 721	174 945	— 5 224	$-2\frac{98}{100}$

The last case of the 320-foot span is probably a misprint. It is scarcely possible that a rational formula, which agrees so closely with practice through such a wide range, should suddenly drop out on a single case while agreeing well with the other two cases of the same example,

and with all the other examples. We expect to hear from Mr. Pegram that there is some misprint or some misconception on our part as to data or construction. The variation above and below the actual is no more than would be due to variations in design and fluctuating unit stress. An absolutely exact formula, if such were possible, might be expected to show such variations. The column of differences is a better criterion than that of percentages. About one ton is the greatest error up to 320 feet span, and there about six tons only, while the smallest difference runs down to thirty-two pounds.

The formula follows the change of depth in the last example, and also follows the change of loading in all the examples. It will be interesting to see how near, by the use of our Table No. 5 and the formulas for wind bracing, we can check Mr. Pegram's *total weight of iron*. Our formulas for wind bracing are based upon very liberal practice, and give results too large in many cases of light bracing which are so common.

Thus in the first example, 104-foot span, 17½-foot panel lengths, we have from Table No. 5, $w_2 = 349$. Subtract 200 for rails, etc., and we have 149 pounds per foot per truss of iron, or $149 \times 2 \times 104 = 30\,992$ pounds for stringers and cross-girders. For wind bracing our allowance is $77 \times 104 = 8\,008$. For truss we have already 23 525, or 47 050 for both trusses. Total weight of iron is then 86 050 pounds, or, adding 3 600 pounds which Mr. Pegram gives for bed-plates, end-shoes and rollers, 89 650 pounds. The actual weight given by Mr. Pegram is 90 555 pounds, or about one per cent. of difference only. In this way we have the following tabulation:

Span	Total Weight of Iron by Formula.	Actual Weight.	Differences.	Per Cent. of Difference.
104	89 650	90 555	− 905	-1
	92 052	93 050	− 998	$-1\frac{7}{10}$
	93 260	96 957	− 3 697	$-3\frac{9}{10}$
150	157 050	154 427	+ 2 623	$+1\frac{8}{10}$
	162 426	163 302	− 876	$-0\frac{5}{10}$
	165 826	177 190	− 11 364	− 6
201½	276 100	267 410	+ 9 690	+ 3
	290 846	286 055	+ 4 791	$+1\frac{6}{10}$
	302 860	304 459	− 1 599	$-0\frac{5}{10}$
320	788 610	778 552	+ 10 058	+ 1
	841 108	837 387	+ 3 721	$+0\frac{4}{10}$
	918 670	946 031	− 27 361	− 2
255½	506 801	508 415	− 1 614	$-0\frac{3}{10}$

Of course the different loading in the several cases affects the floor system, and as we take our own floor system in every case, some fluctuation is to be expected. Still the results are very close, and serve to inspire confidence in our formulas. We believe that such deviations as occur in designing, which a formula cannot follow, would account for our variations. In every example there is at least one case where our deviation from actual amount is not much over one to two tons.

Mr. W. M. Hughes, M. Am. Soc. C. E., has given in the Discussion in the February number for 1886, a list of executed examples. It is difficult to make a satisfactory test of them, because the value of μ used, and the value of $w_1 + w_2 + w_3$ actually employed are unknown. We shall assume $\mu = 8\,000$, and take w_2 and w_3 as given by our Table No. 5 and formulas for wind bracing. For w_1 the values we take are

Span = 50	61	75	84	105	120	135
$w_1 = 1\,700$	1 476	1 200	1 200	1 000	1 000	1 000
Span = 165	184	200	150	142		
$w_1 = 1\,000$	1 000	1 000	2 000	2 000		

The last two spans, 150 and 142 feet, are double track. All are single intersection except the three spans 165, 184 and 200 feet, which are double intersection and wood stringers. These values of w_1, I judge from the formulas, ought to be near those taken by Mr. Hughes. We have then for $w_1 + w_2 + w_3$, the values given in the following tabulation, taking for the wood stringers 50 pounds per foot. If it should happen that these are near the actual values, the test would be very satisfactory, as it would show that from our formulas for weight we have been able to hit the actual loading. The values of w_1 given above are small for present practice, but conform to the old rule of a ton a foot for each track for spans over 100 feet, and may, therefore, we suspect, be not far from correct. If we are right in this, it is because of these light loadings that Mr. Hughes' cases differ so much from the results of Mr. Pegram's formula. We would call attention to the close agreement of the double-track spans, for which Mr. Hughes *found* 51 *and* 59 *per cent. deviation* from Mr. Pegram's formula. The weight for the double intersection spans 165, 184 and 200, includes the wood stringers.

Span in Feet.	Depth in Feet.	No. of Panels.	Panel length in Feet.	$w_1 + w_2 + w_3$	Formula Weight of one Truss per Foot.	Formula Total Weight.	Actual Weight.	Difference.	Per cent. of difference.
50	6¼	4	12½	2 064	147	31 100	31 360	— 260	— $0\frac{8}{10}$
61	8	4	15¼	1 844	153	39 235	39 294	— 59	— $0\frac{15}{100}$
75	21	5	15	1 581	168	52 350	51 157	+ 1 193	+ $2\frac{3}{10}$
84	21	6	14	1 583	192	63 000	62 875	— 125	— $0\frac{2}{10}$
105	20	7	15	1 388	208	83 160	78 476	+ 4 684	+ $5\frac{9}{10}$
120	22.5	8	15	1 391	245	104 640	103 094	+ 1 546	+ $1\frac{5}{10}$
135	22.5	9	15	1 404	281	130 950	124 015	+ 6 935	+ $5\frac{6}{10}$
165	27	11	15	1 276	315	162 030	165 725	— 2 695	— $1\frac{6}{10}$
184	28	12	15⅓	1 287	358	200 560	198 351	+ 2 209	+ $1\frac{11}{100}$
200	28	13	15.385	1 293	397	236 000	240 241	— 4 241	— $1\frac{76}{100}$
150	28	7	21½	2 815	420	250 500	242 930	+ 6 570	+ $2\frac{6}{10}$
142	27	7	20⅓	2 806	399	228 620	235 700	— 7 080	— 3

The agreement is very close, but, as we have said, we can base no conclusion upon it, unless it should appear that the values of $w_1 + w_2 + w_3$ are very nearly those actually used in dimensioning. Any change in our formula (3) due to individual design should be made in the values of 45 and 202, or in the value μ, or both. We would ask Mr. Hughes to test our formula by the actual data, and see whether, with such change, if necessary, it will not check his results.

Professor J. A. L. Waddell, M. Am. Soc. C. E., in "Memoirs of the Tôkiô Daigaku," No. 11, has given a table of weights of bridges of various lengths. The strut formula used by him is that of C. Shaler Smith, M. Am. Soc. C. E., the numerator of which, instead of being constant and equal to 8 000, is variable and given by

$$\frac{40\,000}{4 + \frac{1}{20}\frac{\text{length}}{\text{least diameter}}} \quad \text{or} \quad \frac{10\,000}{1 + \frac{1}{80}\frac{\text{length}}{\text{least diameter}}}$$

It is difficult to satisfactorily test our formula by Professor Waddell's results, without knowing in each case the actual value of μ adopted, and it may be that with these actual values we should need to change 45 and 202. We must leave the test therefore to him. We have tested our formula by his results, however, by assuming for the ratio of

$\frac{\text{length}}{\text{least diameter}}$, the expression $\frac{\sqrt{p^2 + d^2}}{\frac{82}{80}}$ for spans less than 200 feet,

and $\frac{\sqrt{p^2 + d^2}}{\frac{75}{80}}$ for spans above 200 feet. This gives for μ the values

$\frac{10\,000}{1+\frac{\sqrt{p^2+d^2}}{62}}$ and $\frac{10\,000}{1+\frac{\sqrt{p^2+d^2}}{75}}$ With this single change in our formula, intended to hit, as near as may be, the value of μ actually used, we have the following results:

Span in Feet.	Depth in Feet.	No. of Panels.	$w_1+w_2+w_3$ As Given.	w_4 By Formula.	w_4 As Given.
S.I. 80	16	5	1 617	169	169
" 100	20	5	1 616	205	205
" 120	21	6	1 522	248	235
" 140	23	7	1 449	295	294
" 160	24	8	1 399	349	349
" 180	27	8	1 345	395	394
D.I. 180	30	8	1 423	380	392
S.I. 190	28	9	1 323	438	437
D.I. 190	32	9	1 434	423	430
" 210	34	10	1 432	490	486
" 230	38	10	1 387	546	524
" 270	43	12	1 326	673	678
" 300	46	13	1 135	756	798

If it should turn out that we have hit reasonably near the actual values of μ used, and also that where the largest deviations occur, the actual value μ gives better results, the value of our formula would be very strongly established. Here, at all events, is a general formula entirely rational, which checks very closely Professor Waddell's results. As the same formula also gives Mr. Pegram's results, and apparently Mr. Hughes' also, it would seem to be in accord with fact. Professor Waddell will, we hope, give a few hours to the adjustment of the formula to his data, and report his results.

In his report on the "Iron Railway Bridge Across the Mississippi," Van Nostrand, 1869, Mr. T. C. Clarke, M. Am. Soc. C. E., has given in detail the weights of several spans. These spans are of old construction, upper chords cast-iron, short panel lengths, small live load, light floor and wind bracing. They are all double intersection, Phœnix post and chords.

Our weights ought then to be in excess of actual for long spans. It will be interesting, however, to find how close we can check. In all the spans, $w_1 = 1\,250$, $w_2 = 220$, $w_3 = 32$. The stringers are wood, and weight of stringers with track, ties, etc., 150 pounds per foot per truss. The weights given are iron-work only. We take $\mu = 8\,000$.

Span.	Depth.	No. of Panels.	Panel Length.	$w_1 + w_2 + w_3$.	Formula Weight of one Truss per Foot.	Formula Total Weight.	Actual Weight.	Differences.	Per cent. of Difference.
154	22	14	11	1 500	367	147 224	148 592	− 1 368	$-0\frac{92}{100}$
$197\frac{1}{2}$	24	16	$12\frac{1}{3}$	1 500	487	233 248	224 400	+ 8 848	$+3\frac{9}{10}$
247	26	19	13	1 500	656	375 440	347 126	+ 28 314	$+8\frac{16}{100}$

The constants 45 and 202 and μ could easily be changed so as to give the actual weights more accurately. The check under the circumstances is, however, perfectly satisfactory.

In contrast with these results let us take one of the 200-foot spans of the Plattsmouth Bridge, single intersection, span, 200 feet, $d = 30$, $N = 8$, $p = 25$. We have from Table No. 4, $w_1 = 1\,460$, from Table No. 5, $w_2 = 376$, $w_3 = 52$. Then from our formula, for $\mu = 8\,000$, we have $w_4 = 488$. The weight of track, etc., is 230 pounds per foot per truss. Hence, total weight of iron = 274 400. The actual weight is 270 387. Difference, + 4 013 pounds, or $+ 1\frac{46}{100}$ per cent.

We are, we think, justified by the preceding, in asserting that it is possible to produce a formula, based on theoretical considerations alone, of practical value. That such a formula is not necessarily very complicated, and the constants it contains are easily supplied. That such a formula checks well on actual weights, and the influence of the form of truss upon the weight is properly taken into account. We may, we think, go still further. If our formula is as accurate as these comparisons would seem to indicate, then, since it is rational in form, it ought to be of service in determining *best depth and panel length*, as also in deciding between trusses of different types.

Economic Depth.

We see from Table No. 6 that A consists in general of a coefficient multiplied by p^2 plus another coefficient multiplied by d^2. Let us call these coefficients α and β, so that in general $A = \alpha p^2 + \beta d^2$, where the values of α and β are given for any number of panels by Table No. 6. Thus for five panels, for single intersection, $\alpha = 63$ and $\beta = 23.6$, and so on.

Then we have at once from formula (3),

$$\text{economic depth in feet} = \frac{l}{N}\sqrt{\frac{\alpha + \dfrac{45\,\mu}{(w_1 + w_2 + w_3)\,p}}{\beta + \dfrac{202\,\mu}{(w_1 + w_2 + w_3)\,p}}} \qquad (4)$$

where l = span in feet, N = number of panels, μ = numerator of the strut formula used, p = panel length in feet.

w_1 = equivalent uniform load per foot per truss due to the live load.

w_2 = load per foot per truss due to stringers, cross-girders and floor.

w_3 = load per foot per truss due to weight of wind bracing.

α and β are the coefficients of p^2 and d^2 in Table No. 6.

By the use of formula (4) and Tables Nos. 5 and 6, we may then determine, in any case, not only the best depth, but also the best number of panels, or the best panel length.

Let us take as an illustration the first example of Mr. Pegram, viz.: Through span = 104 feet, single intersection, $\mu = 8\,000$. From Table No. 4 we have $w_1 = 1\,633$; for the wind bracing we have $w_3 = 6.43\,N$; for depth between 12.5 and 24; and $w_3 = 5.77\,N$ for depth above 24. The value of w_2 we take from Table No. 5, according to the panel length.

Thus for $N = 4$, we have $p = 26$, $w_2 = 380$, $w_3 =$ about 26, and $w_1 + w_2 + w_3 = 2\,039$. Then from formula (4) we have $d = 25.5$, and from formula (3) $w_4 = 207$. For four panels then, each truss weighs 207 pounds per foot. But if we subtract $w_1 = 1\,633$ from 2 039, we have $w_2 + w_3 = 406$, and if we subtract from this 200 pounds for the rails, etc., we have 206 for iron in the floor, for one truss. The total weight of iron then is 207 + 206 = 413 pounds per foot per truss, or 826 pounds per foot in all.

If we make the calculation thus, for five and six panels we have the following results: Span, 104, single intersection.

				w_4.	Total Weight of Iron per Foot.
$N = 4$	$p = 26$,	$w_1 + w_2 + w_3 = 2\,039$	$d = 25.5$	207	826
$N = 5$	$p = 20.8$,	$w_1 + w_2 + w_3 = 2\,026$	$d = 24$	216	818
$N = 6$	$p = 17\frac{1}{3}$,	$w_1 + w_2 + w_3 = 2\,020$	$d = 20$	234	842

We see at once from this example, that as the number of panels diminishes, or the panel length increases, the truss grows lighter, but the floor system heavier. There is, then, a best number of panels, in this case five, for which the total weight is a minimum.

Mr. Pegram has taken for this case six panels, and a depth of 24 feet. We find the same depth, but a longer panel length, viz., 20.8 in-

stead of 17½ feet. If we were to take six panels, as Mr. Pegram does, then according to our formulas, a depth of 20 feet would be better than 24 feet, and this, we think, will be substantiated by actual design.

Our formulas, therefore, not only furnish us with an accurate estimate of the weight, but are a valuable aid in designing. The best depth for the same span, 104 feet, if double intersection, for 5 panels is $d =$ 23.5 feet, while for Warren girder, 5 panels, we have $d = 20$ feet. The corresponding weights are $w_4 = 208$, and $w_4 = 214$, and total weight, 802 and 814 pounds per foot. For this case and loading then, we see that the double intersection comes out the lightest, and Warren and single intersection about the same, for best dimensions. It would seem then that an intelligent application of our formulas will enable us not only to find the best dimensions and weight for any given style of truss, but to choose between different styles.

For practical and constructive reasons. it would appear best to limit the length of panel to about 25 feet, in case our formulas should lead to a greater length, and for the same reasons to limit the depth for double intersection with posts pinned at center, to 50 feet. Within these limits our method agrees well with the best practice, and confirms the tendency to long panels which is so marked a feature of the latest practice.

Thus, let us take for further illustration Mr. Pegram's second example: through span = 150 feet, double intersection, $\mu = 8\,000$.

We have for

	w_4	Total Weight of Iron per Foot.
$N = 9,\ p = 16\frac{2}{3},\ w_1 + w_2 + w_3 = 1\,992;\ d = 27.8$	327	1 062

Mr. Pegram takes $d = 25$ and gets $w_4 =$ about 325 for a load of 1 844, a result which is also given by our formula.

Careful designing will, I think, justify 27.8 feet as a better depth if 9 panels are used. But we may go further, and say that a still better result would be obtained *by greater panel length.*

Thus we have for

	w_4	Total Weight of Iron per Foot.
$N = 7,\ p = 21^2,\ w_1 + w_2 + w_3 = 1\,994;\ d = 31.8$	301	1 014
$N = 6,\ p = 25,\ w_1 + w_2 + w_3 = 2\,002;\ d = 34.4$	286	1 000

It would seem then that six panels and depth of about 34 feet would be better.

A change in the depth will not much affect the weight, however. Mr. Pegram insists upon this, and our formula confirms it. Thus, if we take $d = 28$, instead of 34.4, we have for $N = 6$, $w_4 = 293$, and total weight = 1 014 instead of 1 000. This is better than Mr. Pegram's proportions, and we can say quite confidently that a longer panel length in this case would be better. In opposition to Mr. Pegram then, we would say that a change in panel length does affect the total weight. If it did not, the tendency to long panels of our present practice would have no justification. The depth also is not a matter of entire indifference, except within somewhat narrow limits.

Our formula indicates also that the depth should be greater for double than for single track. Thus we have for double track

		w_4	Total Weight of Iron per Foot.
$N = 6$, $p = 25$, $w_1 + w_2 + w_3 = 4\,034$;	$d = 40$	458	1 832

If we take 28 feet depth in the first case, we should take about 33 in the case of double track. The formula depths are 34.4 in one case and 40 in the other

For single track, single intersection, and $N = 6$, we have $d = 31$ and $w_4 = 307$, total 1 042. The single intersection should be of less depth than the double, and the total weight is greater.

For Warren girder, single track, $N = 6$, we have $d = 28$ and $w_4 = 308$. This comes out as before, about the same as single intersection, best dimensions taken for both. The economic depth for the Warren is less than for either single or double intersection.

The influence of a change in panel length only is shown by the following, for double intersection, single track:

				w_4	Total Weight of Iron per Foot.
$N = 5$	$p = 30$	$w_1 + w_2 + w_3 = 2\,014$	$d = 34.4$	270	992
$N = 6$	$p = 25$	$w_1 + w_2 + w_3 = 2\,002$	$d = 34.4$	286	1 000
$N = 7$	$p = 21\frac{3}{7}$	$w_1 + w_2 + w_3 = 1\,994$	$d = 34.4$	304	1 020

We see that while the weight of truss diminishes as the number of panels diminishes, the floor system increases. If we limit the panel length to 25 feet, as we have recommended, three trials, as in the case of the 104-foot span, will in any case determine the best number of panels and best depth. If this depth runs too great, it may be somewhat reduced without affecting the result appreciably, but the best number of panels will hold good.

Again, for Mr. Pegram's 320-foot span, double intersection through truss, he takes 16 panels and a depth of 34 feet. For his loading and dimensions, our formula, as we have seen, closely gives his weight. But if we try for 15 and 16 panels, we have

$N = 15$	$p = 21\frac{1}{2}$	$w_1 + w_2 + w_3 = 1\,753$	$d = 53$
$N = 16$	$p = 20$	$w_1 + w_2 + w_3 = 1\,753$	$d = 51.3$

Limiting the depth, however, to 50 feet, we have for

				w_4	Total Weight of Iron per Foot.
$N = 14$	$p = 22.857$	$w_1 + w_2 + w_3 = 1\,750$	$d = 50$	866	2 372
$N = 15$	$p = 21\frac{1}{2}$	$w_1 + w_2 + w_3 = 1\,753$	$d = 50$	835	2 316
$N = 16$	$p = 20$	$w_1 + w_2 + w_3 = 1\,753$	$d = 50$	842	2 330

We have then 15 for the best number of panels instead of 16, and as to the depth, we venture to say that most any depth between 45 and 50 will be found better than the depth of 34 feet assumed by Mr. Pegram. Mr. Pegram's weight of truss is $w_4 = 937$.

We have given enough of illustration to show that if our method and formulas are at all reliable, they will go much further than merely to afford a good estimate of weight alone. Being rational in form, they show the relative influence of all the data which enter into the problem of weight, and serve to settle the important points of proper proportions and best design. The formulas can be applied under any specifications and loading, to any style of truss for which A can be made out.

Mr. Charles Bender, in his recent work, "Principles of Economy in the Design of Metallic Bridges," has made out the value of $\frac{A}{d}$ for many cases, and from the general discussion of minimum values of this quantity, deduces several important relations and rules.

Following this lead, and determining A in similar manner, I have been able to take more or less perfectly into account the *influence of a varying unit stress in the various members, and the influence of the stiffening material required by long struts.* The apparent neglect of these important elements would be the main criticism we should offer upon the very neat and interesting discussion in the work alluded to. Thus Mr. Bender's formula for weight, as we understand it, would take the form

$$w_4 = \frac{w_1 + w_2 + w_3}{\frac{a}{A}\frac{R}{d} - 1}$$

where a would be a coefficient of design and R the *mean unit stress for all members.* We have been unable from this formula to get any re-

sults which would check actual designs through any wide range, while the depths given by it are entirely inadmissible, wholly, as we believe, owing to the influence of long struts, and the varying unit stress they call for. Comparing this formula with our formula (3), it will be seen that in the term $\frac{\mu\,(45\,p^2 + 202\,d^2)}{(w_1 + w_2 + w_3)\,p}$, lies the influence of these elements. Any one wishing to test our formulas upon designs of his own, should simply adjust it in the coefficients 45 and 202 to suit his practice, using for μ and $w_1 + w_2 + w_3$, the exact values actually employed in the design.

Limiting Length.

It will be of interest to point out the significance of the entire term in our formula (3)

$$\frac{3.6\,\mu\,d}{A + \frac{\mu\,(45\,p^2 + 202\,d^2)}{(w_1 + w_2 + w_3)\,d}}$$

If we denote by L the limiting length of girder, or that length for which it will just support its own weight, w_4, we have

$$w_4\,L = (w_1 + w_2 + w_3 + w_4)\,l$$

or

$$w_4 = \frac{w_1 + w_2 + w_3}{\frac{L}{l} - 1}$$

Comparing this with formula (3), we see at once it is identical, and we have the limiting length of girder

$$L = \frac{3.6\,\mu\,d\,l}{A + \frac{\mu\,(45\,p^2 + 202\,d^2)}{(w_1 + w_2 + w_3)\,p}}$$

Thus, in Mr. Pegram's first case, span 104, $d = 24$, $N = 6$, single intersection, we have for $w_1 + w_2 + w_3 = 1\,820$, $L =$ about 940 feet. For Mr. Pegram's 320-foot span, $N = 16$, $d = 34$, double intersection, we have for $w_1 + w_2 + w_3 = 1\,605$, $L = 894$. If we should assume that the limiting length were constant for any style of truss, say not far from 940 feet for single and double intersection, we should have at once the very simple formula

$$w_4 = \frac{w_1 + w_2 + w_3}{\frac{940}{l} - 1}$$

How far such a formula would go in checking Mr. Pegram's results, the following tabulation will show:

Span.	$w_1 + w_2 + w_3$	Formula Weight of One Truss.	Actual Weight of One Truss.	Per Cent. of Difference.
104	1 820	23 505	23 658	$-0\frac{64}{100}$
	2 000	25 896	24 458	$+5$
	2 090	27 040	25 074	$+7$
150	1 675	47 700	46 032	$+3$
	1 844	52 500	49 628	$+5$
	1 950	55 500	52 627	$+5$
201.5	1 565	85 637	85 131	$+0\frac{6}{10}$
	1 776	97 123	93 626	$+3$
	1 946	106 593	100 283	$+6$
320	1 605	265 920	273 671	-2
	1 798	297 920	299 718	-6
	2 088	345 920	302 206	?
255.5	1 943	185 237	185 558	$-0\frac{17}{100}$

Here, then, is a single, simple formula made "off-hand," which satisfies well the entire range of all Mr. Pegram's examples quite as well in fact as his own series of formulas, with their carefully determined constants. Probably a little pains would determine a value for L which would give even better results. The formula as it is, judged by agreement alone, is about as good as Mr. Pegram's series, and with two or three values of L to match his coefficients, would easily be better. As it is, if no great accuracy is required, it is good enough for general purposes, as, for instance, for an estimate of weight preliminary to calculation of strains. It is also rational in form so far as it goes, but its accuracy depends upon the assumption that the limiting length is practically constant. Only in so far as this assumption is correct, is this formula reliable.

The comparison just given shows how far this assumption is practically correct. It is questionable whether, in the close competition of the day, any engineer would base a bid entirely upon a formula, and we doubt whether Mr. Pegram would so use his own formulas, in view of the variations shown by his tables of results. A formula which is liable to vary by 4 per cent. or more, would be of little value for such purpose. But that same close competition renders of great interest the questions of best depth and panel length, and all points bearing upon good design. A sound, rational formula, which gives reliable estimate of weight, ought to have a valuable bearing upon such questions, while empirical

formulas are, for such purposes, worthless; it is in this that we consider the main value of our formulas to lie; it is from this point of view that the present article is written; and from this point of view that we offer the present method and formulas for discussion.

Demonstration of the Formulas.

The deduction of formulas (1), (2), (3), and (4) is simple, and will take but little space in conclusion.

As the weight depends upon the areas, and these are actually determined by statical calculation, any rational formula must be based at bottom upon such statical calculation, and a general statical calculation ought to furnish a reliable formula.

Let us first deduce formulas (1) and (2) for plate girders.

Let the total external load be represented by W, and let W_1 be the weight of the girder itself. Let l = the span in feet, and d = the depth in inches. Let R = the working stress per square inch in the flanges.

Then, since the total load is $W + W_1$, we have for the moment at the center $\frac{(W + W_1) \times 12\ l}{8}$ inch pounds.

If we divide this by the depth in inches, d, we have the flange strain. Dividing the flange strain by R, we have the flange area. The area of both flanges then will be $\frac{(W + W_1)\ 12\ l}{4\ R\ d}$. If we take the thickness of the web at t inches, its area is $d\ t$ square inches. We have then for the sectional area of the girder $\frac{(W + W_1)\ 12\ l}{4\ R\ d} + d\ t$ square inches.

If we multiply this by $\frac{10}{3}$ we have the weight of one foot in length. The weight of l feet then is

$$W_1 = \left[\frac{(W + W_1)\ 12\ l}{4\ R\ d} + d\ t\right]\frac{10}{3}\ l$$

From this, taking the thickness t at $\frac{1}{2}$ inch, we have

$$W_1 = \frac{12\ W\ l^2 + 2\ R\ l\ d^2}{1.2\ R\ d - 12\ l^2} \qquad (1)$$

Differentiating, and putting first differential equal to zero, we have

$$\text{economic depth in inches} = \frac{10\ l^2}{R} + \sqrt{\frac{6\ W\ l}{R} + \left(\frac{10\ l^2}{R}\right)^2} \qquad (2)$$

These are formulas (1) and (2) already given for plate girders.

Let us now deduce (3) and (4).

Take, for illustration, a Warren girder having N panels, and consider it loaded at every lower panel point with the full panel load $(w_1 + w_2 + w_3 + w_4)\, p$. Since there are $N - 1$ such loads, the reaction is $\frac{(w_1 + w_2 + w_3 + w_4)\,(N-1)\,p}{2}$.

The strain in the first lower panel is the reaction multiplied by the half-panel length and divided by the depth. If this strain is divided by the stress per square inch for tension R_t, we have the area in square inches. Multiply this area by $\frac{10}{3}$ and by the panel length p, and we have the weight of the first lower panel.

We have then for the weight of the first lower panel

$$\frac{10\,(w_1 + w_2 + w_3 + w_4)\,p_3}{12\,R_t\,d}\,[N-1].$$

In a similar way we find the weight of each lower panel, and thus obtain the following values:

Weight of 1st lower panel	$\frac{10\,(w_1 + w_2 + w_3 + w_4)\,p^3}{12\,R_t\,d}$				$[N-1]$
"	2d	"	"	"	$[3\,(N-1) - 2]$
"	3d	"	"	"	$[5\,(N-1) - 8]$
"	4th	"	"	"	$[7\,(N-1) - 18]$

and so on.

Summing up by series, we can easily find the weight of N lower panels, or the whole lower chord:

$$\frac{5\,(w_1 + w_2 + w_3 + w_4)\,Np^3\,(N^2-1)}{18\,R_t\,d}$$

Since $Np = l =$ the span, the weight per foot per truss of the lower chord is

$$\frac{5\,(w_1 + w_2 + w_3 + w_4)\,p^2\,(N^2-1)}{18\,R_t\,d}$$

In a precisely similar manner we find for the weight of the upper chord per foot per truss:

$$\frac{5\,(w_1 + w_2 + w_3 + w_4)\,p^2\,(N^2-1)}{18\,R_c\,d}$$

where R_c is the stress per square inch for compression. We thus, it

will be observed, keep the stress per square inch for tension and compression separate.

For the braces the secant of the angle they make with the vertical is

$$\sec.\Theta = \frac{\sqrt{\frac{p^2}{4} + d^2}}{d} = \frac{\sqrt{p^2 + 4d^2}}{2d}$$ We have then for full loading the strain in the first tie, $\frac{p\,(w_1 + w_2 + w_3 + w_4)\,\sqrt{4d^2 + p^2}}{2d}$ $[N-1]$

Dividing by R_t for the area, and multiplying by the length $\frac{\sqrt{4d^2 + p^2}}{2}$ and by $\frac{10}{3}$, we have:

Weight of 1st tie $\frac{10\,(w_1 + w_2 + w_3 + w_4)\,(4d^2 + p^2)\,p}{12\,R_t\,d}$ $[N-1]$

" 2d " " $[(N-1)-2]$

" 3d " " $[(N-1)-4]$

" 4th " " $[(N-1)-6]$

and so on.

The weight of N ties is then $\frac{5\,(w_1 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N^2\,p}{12\,R_t\,d}$

and the weight per foot of the ties per truss is

$$\frac{5\,(w_1 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N}{12\,R_t\,d}$$

In similar manner for the struts we have

$$\frac{5\,(w_1 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N}{12\,R_c\,d}$$

where R_c is the stress per square inch for compression.

The total weight per foot is, then,

$$w_{-4} = \frac{5\,(w_1 + w_2 + w_3 + w_4)}{18d}\left[\frac{(N^2-1)\,p^2}{R_t} + \frac{(N^2-1)\,p^2}{R_c} + \frac{1.5\,N\,(p^2 + 4d^2)}{R_t} + \frac{1.5\,N\,(p^2 + 4d^2)}{R_c}\right]$$

For the sake of brevity let us write

$$w_4 = \frac{5\,(w_1 + w_2 + w_3 + w_4)}{18d}\left[\frac{T}{R_t} + \frac{C}{R_c} + \frac{S}{R_s}\right]$$

where T refers to the lower chord and ties, or all tension members, C to

the upper chords, and S to the struts; hence $T=(N^2-1)\,p^2+1.5\,N\,(p^2+4d^2)$, $C=(N^2-1)\,p^2$, and $S=1.5\,N\,(p^2+4d^2)$.

We have then

$$w_4=\frac{w_1+w_2+w_3}{\dfrac{3.6\,d}{\dfrac{T}{Rt}+\dfrac{C}{R_c}+\dfrac{S}{R_s}}-1}$$

Now the strut formula in common use is, for the upper chords,

$$R_c=\frac{\mu}{1+\dfrac{p^2}{c\,r_1^2}}$$

and for the struts

$$R_s=\frac{\mu}{1+\dfrac{p^2+4\,d^2}{4\,c\,r_2^2}},$$

where $\mu=8\,000$ for iron, and r_1 and r_2 are least radii of gyration of the cross-section, and c a constant depending upon end conditions.

We have then, inserting these values of R_c and R_s,

$$w_4=\frac{w_1+w_2+w_3}{\dfrac{3.6\,\mu\,d}{\dfrac{\mu}{R_t}\,T+C+S+\dfrac{Cp^2}{c\,r_1^2}+\dfrac{S\,(p^2+4\,d^2)}{4\,c\,r_2^2}}-1}$$

Now R_t is, on the average, about 9 000 pounds, and $\mu=8\,000$ pounds. We shall make but slight error in taking $\dfrac{\mu}{R_t}=1$. Such slight error as there is, is partially balanced by the fact that we have found the weight of web for full load instead of partial live load and full dead load. If then we put $\dfrac{\mu}{R_t}=1$ and $T+C+S=A=p^2\,(2\,N^2+3\,N-2)+12\,N\,d^2$, we have

$$w_4=\frac{w_1+w_2+w_3}{\dfrac{3.6\,\mu\,d}{A+\dfrac{Cp^2}{c\,r_1^2}+\dfrac{S\,(p^2+4\,d^2)}{4\,c\,r_2^2}}-1}$$

Now for the upper chords the square of the radius of gyration, r_1^2, should increase as C increases, and also as the panel load increases, and diminish as μ increases. So also for r_2^2. We ought to have then, approximately,

$$r_1^2 = \frac{(w_1 + w_2 + w_3)\, p\, C}{a^1\, \mu}, \quad r_2^2 = \frac{(w_1 + w_2 + w_3)\, p\, S}{b^1\, \mu}$$

where a^1 and b^1 are empirical constants. This step in the development is the one to be specially criticised. Upon it depends the accuracy of our formula. The results of the application of the formula are the best justification of the accuracy of this reduction, by which the strut formula is introduced. Inserting these values, we have finally, for the weight per foot per truss

$$w_4 = \frac{w_1 + w_2 + w_3}{\dfrac{3.6\, \mu\, d}{A + \dfrac{\mu\,(a\, p^2 + b\, d^2)}{(w_1 + w_2 + w_3)\, p}} - 1} \qquad (3)$$

where a and b are constants to be determined by actual cases and individual design, and for Warren girder, $A = p^2\,(2\,N^2 + 3\,N - 2) + 12\,N\,d^2$. We find by comparison with actual examples $a = 45$, $b = 202$.

We can easily determine A for other types of truss, as given in the text.

Replacing A by its general value $\alpha\, p^2 + \beta\, d^2$, differentiating and putting first differential equal to zero, we have

$$\text{economic depth} = \frac{l}{N}\sqrt{\frac{\alpha + \dfrac{45\,\mu}{(w_1 + w_2 + w_3)\, p}}{\beta + \dfrac{202\,\mu}{(w_1 + w_2 + w_3)\, p}}} \qquad (4)$$

DISCUSSION ON
FORMULAS FOR THE WEIGHTS OF BRIDGES.

J. A. L. WADDELL, M. Am. Soc. C. E.—It is to me a matter of deep regret that I am unable to spare the time to discuss Professor DuBois' valuable paper in the manner that he suggests.

Nothing but sheer necessity would prevent me from complying with such a request; but at this season there is no time to spare in the office of a bridge engineer, the great call for bridges making it necessary to work day and night.

However, as far as my table of weights is concerned, further adjustment of Professor DuBois' formula seems unnecessary, judging by the accuracy of most of the results in his table.

Professor DuBois' method of dividing bridge weights into those for floor system, lateral system and trusses, is the most rational that could be employed; for the weights of these portions are almost independent of each other.

The weight of floor system depends upon the panel length and the amount and distribution of weight on engine wheels.

The weight of lateral system depends upon the panel length and the wind pressure assumed.

The weight of trusses depends upon the panel length and the combined live and dead loads. The system of concentration of wheel loads affects the truss weight but slightly.

There is, however, another very important factor affecting the values of these weights, viz., the specifications by which the designs are prepared. Until there is greater uniformity in general specifications for railroad bridges it is useless to hope for accuracy in formula for weights. The fact that this want of uniformity exists is no credit to our profession; it simply shows that bridge engineers do not sufficiently discuss the subject of bridge-designing. Each one works too much by himself, and consequently fails to benefit by the experience of the rest. In my opinion there ought to be brought about, either through the American Society of Civil Engineers or otherwise, a convention of bridge engineers, comprising all those who are known to have given the subject of designing much study. Plenty of time should be allowed to prepare for this convention, topics for discussion being sent to the committee months beforehand, and published in the engineering journals. Several conventions would probably be necessary in order to arrive at definite conclusions. At the first no decision should be attempted, but the discussions should be taken down in full, published and circulated. At the next meeting the discussions might be continued and recorded as before; then, before adjournment, a committee should be appointed to draft one or more sets of complete general specifications. Finally, at

another meeting, these could be discussed, voted upon and adopted with the necessary modifications.

Such a method of procedure would cost both time and money, but the results would be well worth all the trouble and expense.

A. GOTTLIEB, M. Am. Soc. C. E.—I have read with much interest the paper of Professor DuBois, about determining the weights of bridges, finding the most economical depth of trusses, panel lengths, etc.

I cheerfully give credit to him for the neat, concise, and, I might say, perfect deductions of his formulas, and the great accuracy achieved by them in finding the weight of bridges.

I am sorry to say that the time at my disposal did not permit me to test the formulas myself, on data in my possession, but I accept Professor DuBois' comparisons with Mr. Pegram's tables of weights, and other executed bridges, as undoubtedly correct. The result proves, if anything, that Professor Du Bois has not only a clear understanding of the subject, but has also taken great pains in elaborating it in a scientific manner, with the employment of the least possible number of empirical coefficients or factors.

The questions that present themselves to my mind, and undoubtedly to others, are: What is the practical value of these formulas, or will they ever be used to any great extent? Is there any shorter and more convenient way to obtain the same results?

I think there is, at least as to determining the weights of bridges. For what purpose do we wish to determine these weights? For an approximate estimate to determine the cost of structures the formulas given will do very well indeed, although for truss bridges they are too complicated for such a purpose, and there are probably other ready means on hand for every engineer to use. For actual competition for work, Mr. DuBois himself states few engineers would be willing to use them at the close competition existing. There remains, therefore, principally their usefulness in aiding to find the correct dead weight to be used in calculation of strains in bridges to resist certain given live loads. As far as plate-girders, or cross-girders in truss spans and stringers, are concerned, the formulas are simple enough, but are nothing more or less than the calculation of the actual strains, sections and weights for the girders for a given live load and an assumed general dead load W. For short girders this formula is accurate enough, but must be corrected for long girders, the flange section in the center being assumed to be uniform throughout, which, of course, is not actually the case. The increased amount of stiffness, and perhaps also thickness of web at ends, does not compensate for the deficiency in section of flanges at the ends of long girders, say over fifty feet.

As to the formulas for trusses, it is certainly a very laborious task to find the value of A for each kind or style of truss, which in itself will deter engineers in practical life from using it,

The simpler, quicker, and fully as accurate plan which I would suggest, is to figure the strains for the live load separate from those for the dead load, which latter may be assumed near enough from previous experience; or, if tried by an inexperienced party, by assuming a dead load *ad libitum*, proportion the parts separate for dead-load and live-load strains, and compute the weights of the structure thereby. If the result does not check the assumed dead weight, make the necessary correction, which is very quickly done. The weight of floor, cross-girders and stringers, of course can be ascertained easily beforehand, as suggested by Professor DuBois.

As to the finding of the most economic depth or height of trusses, panel lengths, etc., the formulas are valuable. But I submit for consideration the fact, that ultimately that design is the most practicable which, when performing the same duty, is the cheapest, not always the lightest.

While the saving of material is the first object desired, with a view to proper economy, we must consider also the amount of work required for different designs, and the practicability of obtaining the material of such dimensions as the theoretic formula may prescribe.

PALMER C. RICKETTS, Assoc. Am. Soc. C. E.—It will be admitted that rational formulas deduced for purposes of generalization are more liable to give correct results within the limits considered than those which are purely empiric; and that of rational formulas that one will, within limits, probably be the most satisfactory for general application which takes account of the most of the variable conditions existing. The purpose of such formulas may however be defeated by their complexity, and one purely empiric, or nearly so, may, of course, be more advantageous for general use. The remarks on the plate-girder formula of Professor DuBois are suggested by its application to a number of actual weights of girders built in accordance with first-class modern bridge practice for engines given by the following figures:

| | | |

differing from that given as Class C by Mr. George H. Pegram in his paper published in Vol. XV of the Transactions of this Society, approaches it more nearly than it does to any of the others for which he gives formulas, and it is proposed to compare the weights obtained by the use of his corresponding equation with the ones found from that discussed here. In making the comparison, it is remembered that the formula considered is more general, but the three given by Mr. Pegram differing by a constant would seem to cover most engines used in specifications at present. The following table giving, for plate girder-spans of from twenty to fifty feet, the actual weights and those calculated by the two equations, seems to show that the use of that of Professor DuBois would not be warranted in this case. The length, center to center of bed plates was used in both cases.

PLATE GIRDERS.

Span C to C.	Depth.	Weight.		
		Actual.	DuBois.	Pegram.
Ft. In.	Ft. In.			
20 0	2 0	7 119	6 123	6 903
26 6	2 3	9 480	9 418	10 616
33 6	3 0	15 000	13 826	15 291
36 0	3 0	16 640	15 748	17 064
42 6	3 6	21 944	20 859	22 090
53 6	5 0	31 650	30 538	31 656

A considerable number of other actual weights of spans from fourteen to fifty feet were also compared with those obtained by the use of Mr. Pegram's equation, and were found to agree about as well as those given.

With regard to the claim for accurate economic depth, it seems to me that a formula, deduced as this one is by the use of an equivalent load, and of a cross-section of flange not necessarily the mean, and in which the assumption of a thickness of web greater than that generally used helps to make up for the weight not considered at the ends, can hardly be considered to give an exact economic depth, especially as Professor DuBois truly remarks that a considerable variation in depth makes slight difference in the weight. I regret that I have not sufficient data at hand to apply the formula for truss bridges, but similar remarks would apply to it also. It is not intended here to disparage the results of Professor DuBois, whose ability and knowledge of the subject is well known, but the criticisms apply on account of the difficulty of including, in formula sufficiently simple to render their use general, many of the conditions which enter into the proper design of a bridge.

CHARLES J. MORSE, M. Am. Soc. C. E.—I have found Professor DuBois' paper very interesting, and think his formulas more closely approximate the analytical solution of the much vexed problem of the weights of bridges than any I have seen. I think his method of making separate calculations of floor and bracing preliminary to finding the weights of the trusses is the correct method of procedure for determining an accurate formula, although more tedious of application.

The formula he obtains is, however, only an approximation and not a solution. It comes nearer a solution because it includes, or attempts to include, one more condition of the many involved, viz., the strut formula or varying unit strain.

His formula for stringers is simple and accurate for the flanges and web. The "details," such as end connections, ribs and fillers, rivets, etc., are provided for by assuming the web to be one-half an inch thick for all depths and lengths. The same formula applies to floor beams by changing the uniform live load to suit their conditions of loading and to plate girders, deck or through, after first calculating the bracing and floor systems. Whether this web assumption accurately provides for the details, we have not had time to determine.

The formula for the weight of truss is derived by summing the expressions for the weights of each principal part, but takes no account of details whatever until the strut formula is introduced.

This strut formula is introduced by assuming a value for the square of the radius of gyration, in terms of the panel loads, number of panels, unit strains, etc. The assumption seems a reasonable one, but it may or may not be correct. With the strut formula is introduced a constant to be determined empirically; or rather two constants, one for upper chord-strut formula and the other for intermediate posts. These constants have to provide for not only the special peculiarities of design, but for all details as well, including pins, lattice and tie bars, pin plates, rivets, nuts, etc. It seems to me a clear case of overloaded constant.

I believe the soundness of the formula is vitiated by assuming that the details (which by the way sometimes amount to 25 per cent. of the whole) can be provided for by these constants or that details are a constant factor of the weight. I know in fact that the details are a *varying* percentage of the weight of the principal parts, varying inversely as their weights.

The author derives from his formula for weights of girders and trusses, by the method of maxima and minima, the most economic depths, etc.

This "most economic depth" is based simply on "least weight of material," irrespective of the relative cost of material or shop construction, of good proportioning, or other considerations.

It is interesting and valuable as far as it goes, but unfortunately it stops far too short to be of much practical use. I think such formulas

as Professor DuBois has ingeniously devised are useful for an analytical discussion of general laws governing weights, depths, etc., for hints and suggestions they furnish the designer; but of little practical value for every-day service, because they are not accurate enough for final estimates, and require too much labor for simply finding dead weights to use in calculating strains.

The economic depths determined by the formulas are excessively large, as might be expected. I do not agree with Professor DuBois that 7.7 feet is a proper depth for a 63-foot plate girder, even though that depth gives the least weight.

Considerations of relative prices of materials, of lateral strength, of shop cost, etc., tend to reduce this depth considerably.

To get some some idea of the effect of prices of material in modifying economic depth, I have taken Professor DuBois' formula for simple stringers and introduced constants for the prices of angles, plates, etc. Using his notation, his formula for weight of simple girder is

$$\left[\frac{(W + W_1)\,12\,l}{4\,R\,d} + d\,t\right]\frac{10}{3}\,l$$

the first term of which gives the weight of the flanges and the second the web and details.

Let a = price per pound for angles (flanges assumed to be all of angles).

" p = price per pound of web plates.

Since in assuming $t = \frac{1}{2}$ inch, the weight of all end connections, rib and fillers, rivets, etc., are included with the web, it will be a close approximation to assume that the proportion of plate to other materials included with web is as 3 to 1, and that the average price of the details, viz., ribs, fillers, rivets, etc., will be that of angles.

Hence let $$\tfrac{3}{4}\,p + \tfrac{1}{4}\,a = m = \frac{3p + a}{4}$$

Making $t = \frac{1}{2}$ inch and multiplying each term by its price per pound, we get

$$\text{Total cost} = \frac{10\,(W + W_1)\,a\,l^2}{R\,d} + \frac{5\,m\,d\,l}{3}$$

assuming that (W_1) is constant, since it varies but little for small changes of depth. Differentiating and placing equal to zero, most economical depth for minimum cost of material,

$$\frac{6\,a\,l\,(W + W_1)}{R\,m}$$

Applying this formula and using the values given in Professor DuBois' Table No. 1, and assuming for trial that the price of plates is ten

per cent. greater than angles, we have $a=1$, $p=1.10$, $m=1.075$, and obtain the following results:

l. Panel length in feet.	*W.* Uniform live load per stringer.	*d.* Economic depth for least weight.	*d.* Economic depth for least cost.	Difference per cent.
10	35 100	16.4 inches.	15.7	—4.27
15	58 066	25 8 "	24.9	—3.50
20	70 200	33.0 "	31.8	—3.63
25	84 500	40.7 "	39.2	—3.68
30	95 784	47.5 "	45.9	—3.37

The application of the same formula to an 80-foot plate girder, under same loading, calling (a) the average price of the plates and angles comprising the flanges, and again assuming the wide web plates to cost ten per cent. more than this average, gave the economic depth 101.8 inches, instead of 103.4 inches as given by Professor DuBois.

These results show that the relative price of material as far as girders are concerned, does not cut a very large figure in modifying economic depth, but amounts to enough to be taken into account in an analytical formula. If now an expression for "good practice" could be introduced into the formula, and another variable for "shop cost," and another for "cost of erection," a resulting "best depth" might be obtained. All these conditions must be fulfilled in economical designing, and therefore ought to be contained in any analytical formula proposing to give "the most economical depth."

H. C. Jennings, M. Am. Soc. C. E.—Before proceeding with the discussion of the application of Professor Du Bois' formulas to the determination of the actual weights of plate-girder bridges and their economic depths, I would like to mention a few facts in regard to the original of the 104-foot span upon which the author has based considerable of his comparison of Mr. G. H. Pegram's formula with his own, and to the design of which he takes exception.

The original of this 104-foot span is on the line of the Chicago, Milwaukee and St. Paul Railway, having been built about ten years ago, and was probably one of the first of a style of through truss bridge which has since been extensively adopted by engineers of this country, especially for spans under 200 feet. The point made in the design, and for which especial excellence was claimed, is the riveting of the floor-beams to the posts, above the bottom chord, thus forming, with the top struts and knee-braces, a rigid resistance to any overturning effort at each panel point. In order to accomplish this, 24 feet was the minimum depth of truss permissible to allow the proper clear height for safe passage of trains. It will be seen from this explanation that there were other considerations than that of the least amount of iron in the structure that determined the proper depth of truss to be used.

I will now proceed to the plate-girder formulas, and would present as a basis for what I have to say the following tables.

TABLE No. 1.

Length of girder.	Width C. to C.	Style.	Weight of girders.		Weight of bracing.		Weight of bed plates and bolts.	Total weight.		Per cent. of difference.
			Actual.	By formula.	Actual.	By formula.	Actual.	Actual.	By formula.	
Feet.	Feet.									
41	14	Through	17 035	15 104	3 505	2 750	888	21 428	17 854	—16 7/10
49½	14	"	22 476	22 328	3 832	3 591	997	27 305	25 919	— 5 1/10
60	14	"	33 576	33 324	5 201	4 536	1 053	39 830	37 860	— 5
63	14	"	36 694	35 768	5 387	4 601	1 269	43 350	40 369	— 7
68	15	"	44 160	43 422	7 537	4 709	1 353	53 050	48 131	— 9 2/10

The depths used in the formula were the actual depths as built, and were as follows: For 41-foot span, 42 inches; 49½-foot span, 50 inches; 60-foot span, 60 inches; 63-foot span, 66 inches; and 68-foot span, 70 inches.

TABLE No. 2.

	Length.	Width.	Depth.	Style.	Weight of girders.		Weight of bracing.		Weight of bed plates and bolts.	Total weight.		Per cent. difference.
					Actual.	By formula.	Actual.	By formula.	Actual.	Actual.	By formula.	
	Feet.	Feet.	In.									
a	45	6½	48	Deck	20 236	16 122	1 263	2 808	....	21 599	18 930	—12 3/10
b	30	6½	48	"	8 816	7 396	885	1 994	....	9 701	9 340	— 3 7/10
c	45	9	54	"	20 664	17 020	2 820	2 808	900	24 384	19 828	—18 7/10
d	30	9	42	"	11 284	8 082	2 150	1 944	600	14 034	10 026	—28 6/10

Spans *a* and *b* were figured for flange sections based on the moment of inertia of the net section, with a unit strain from Weyrauch's formula, 10 000 pounds per square inch; spans *c* and *d* were figured with depth back to back of angles and *r* 9 000 pounds per square inch.

Table No. 1 shows a comparison of the actual weights and the formula weights of a few plate-girders built for the Chicago, Milwaukee and St. Paul Railway some years ago, when the live load in use for bridges of this class was 4 000 pounds per lineal foot. The type of these bridges was a through girder with wooden floor-beams supported at their ends by an angle-iron shelf riveted to the lower part of the web.

This floor, with the rails and guard timbers, is assumed to weigh 700 pounds per foot for width, center to center of girders, of 14 feet, and 800 pounds per foot for width of 15 feet.

In Table No. 2 I have shown a comparison with spans proportioned in accordance with our present practice, in which the live load is given by Mr. G. H. Pegram in his Class T, with a percentage added for momentum of 30 per cent. for spans under 25 feet, and, in proportion to length of span, up to 8 per cent., for 80 feet. In all the above girders the web is considered to have no part in resisting flange stress. It will be seen that the formula weights are less than the actual in all cases. Let us now proceed to an analysis of the formulas and see if we can expect from them reliable results.

Considering first the weight of the girder only, it may be said to be composed of the following items, viz.:

First.—Weight of material in the flanges.

Second.—Weight of material in the web.

Third.—Weight of stiffeners, splices, bearing plates, and for girders with wooden floor, shelf-angles.

Professor DuBois by assuming the first item to be uniform throughout the entire length of the girder, and the second considerably in excess of the average practice, presents a formula for the total weight of the girder in which the third item is entirely disregarded. It would appear from this that the reliability of the results from the formula depends on the balancing of excess and deficiency. Can such a formula be termed rational, or relied upon to give accurate weights.

The empirical formula for the weight of the bracing would seem, from the fact that it is entirely independent of width or style of bridge, to be only adapted to some one style of design. This can be illustrated by an example showing the actual weights of bracing for two plate-girder bridges, length of each 60 feet, one deck and the other through, both with wooden floors.

Bracing for 60-foot deck weighs 3 605 pounds.

Bracing for 60-foot through weighs 5 201 pounds.

Again, it will be seen on reference to the application of formula to the 63-foot span in Professor DuBois' paper, that he makes the weight of bracing for that span with iron floor-beams and stringers to be 3 067 pounds; in the table above presented is a 63-foot span, with wooden floor, in which the bracing weighed 5 387 pounds, of which 4 115 pounds was the weight of cross-struts, which would be replaced by the floor-beams in a design with iron-floor system, leaving but 1 272 pounds for the weight of diagonals. Here again it would seem that the method employed by Professor DuBois is but an approximation, and cannot give close results. Were we to consider the total weight of the structure, another item of weight would seem to be neglected, that of bed-plates, expansion rollers (if used), and anchor bolts. In the 60-foot span referred to above, bed-plates and bolts weighed 1 053 pounds.

While agreeing with Professor DuBois in the theory that the formulas for weights of bridges should be rational, I disagree with him in the method of deriving such formula. To produce reliable results by any such process as used by Professor DuBois, the much discussed problem of proper thickness and stiffening of web plates must be introduced. The absence of data on this point on which we can base conclusive deductions, leaves this question open to individual constructions, giving rise, as at present, to considerable variation of opinion as to both the thickness of web and number of stiffeners necessary to a properly designed plate-girder. Now, as this as yet unsolved problem appears to be a stumbling block in the way of a correct formula based on Professor DuBois' methods, can we not find some equally as rational a method that will not call into play this obstacle. Adopting as a method for computing live-load stresses a uniformly distributed load which will produce a center moment equal to that from the worst position of wheel loads, and from this center moment proportioning the material in the flanges according to the law for uniformly distributed loads, the weight of the material in the flanges of a girder should bear a constant relation to the bending moments, which latter can be expressed by a parabolic curve. The weight of the web, stiffeners, bracing and other component parts can be expressed by an equation of a line founded on the summation of the proper functions of length of span, depth of girder, width of bridge and assumed wind load. Having now all the component weights of the structure represented by the equations of two lines, which may, from the analysis of actual bridges, be accurately platted, it but remains to combine the equations into one in which all conditions affecting the weight are represented. I claim no originality for this method of determining bridge weights, but feel convinced that if ever a reliable formula is deduced it will be by this method.

Edwin Thacher, M. Am. Soc. C. E.—As the object of this paper is to furnish formulas which will be of service to bridge-builders in determining the weights of bridges in advance of detailed calculations, and which will give results agreeing closely with such calculations, it is hoped that the author will kindly consider the following remarks.

I am a little in doubt as to what he intends his formula (1) to represent. Judged by his demonstration it represents the weight of the flanges and web of a plate-girder, to which must be added the weight of the miscellaneous iron-work necessary to complete the structure; but in his example of a 63-foot through girder he would appear to make it represent the entire weight of iron-work, exclusive of wind bracing, and perhaps masonry plates, and we presume this latter is his intention.

It is expecting too much of any one formula to be applicable to all specifications, and I can see no reason why the one under consideration should be applicable to any. It assumes:

First.—Flanges of uniform section.
Second.—Half-inch thickness of web.
Third.—Effective depth of girder identical with depth of web.
Fourth.—Length between bearings identical with total length.
Fifth.—No loss from rivet-holes in tension flange; and
Sixth.—No allowance for the effect of web in flange area.

The first assumption is not often correct except in light girders; the second only in very heavy or very shallow ones; the third only in heavy girders; the fourth only for track stringers and floor-beams with riveted connections; the fifth is never correct; and as for the sixth, this is governed by specification. But the writer has never yet been able to understand how a plate can be securely riveted between two angles and not be strained equally with the angles. A formula to be useful must be simple; must consider the conditions about as they are; and must meet the specifications. In the table following formula (1) are compared the formula weights and what are said to be actual weights of girders for spans up to 80 feet in length. I do not understand how these actual weights can agree so closely with the formula, unless either the same methods were used in the calculation as were used in making the formula, or by the chance balancing of errors. It appears, however, to agree surprisingly well in some comparisons made by the writer with actual estimates when the web is omitted in flange area and a liberal provision is made for stiffeners, and it will undoubtedly answer the purpose for approximate estimates or for assuming dead load preliminary to estimating, and this, I believe, is all that the author claims for it.

There is no great difficulty in making simple formulas for the weight of plate-girders, using the same data in about the same way as it would be used in careful estimates, and which will give results agreeing as closely with such estimates as those made by different computers will agree with each other, and if weight and cost only are required, further estimating can be dispensed with.

We will proceed to give such formulas, using the following notation:

l, length of girders from center to center of bearings.
l_1, length of span over all.
p, length of panel.
x, distance from center of girders to center of stringers in through girders.
d, depth of girder between centers of gravity of flanges.
d_1, depth of web.
t, thickness of web.
S, mean stress per square inch in flanges.
A, area in square inches of two angles in either flange, and such flange plates, if any, as extend the full length of girder.
W, weight supported per lineal foot of rail.
w, weight per lineal foot of flanges necessary to support W.
w_1, weight per lineal foot of girder due to web.
w_2, weight per lineal foot of girder due to stiffeners.
a, weight per lineal foot of a pair of stiffeners, including fillers if used.

w_3, weight per lineal foot of span due to all iron-work, exclusive of flanges, web and stiffeners.

N, number of panels.

Thickness of Web.

This is usually governed by the specifications for rivet-bearing, thus:

For longitudinal girders,

$$t = \frac{W l \times \text{pitch}}{24\, d \times (\text{diameter of rivet}) \times (\text{bearing per square inch})} \quad (1)$$

For cross-girders,

$$t = \frac{W p \times \text{pitch}}{12\, d \times (\text{diameter of rivet}) \times (\text{bearing per square inch})} \quad (2)$$

Very few specifications allow less than 12 000 pounds per square inch diametrical bearing, and assuming $\frac{7}{8}$-inch rivets with a minimum pitch of 3 inches, the required thickness of web in inches will be

For longitudinal girders..... $t = \frac{W l}{84\,000\, d}$ (3)

For cross-girders ..... $t = \frac{W p}{42\,000\, d}$ (4)

t will rarely exceed $\frac{3}{8}$-inch in thickness.

Flanges.

Plate-girders may be divided into two classes:

First.—Girders having flanges of uniform section.

Second.—Girders having flange angles of uniform section, and flange plates of uniform strength.

In either class the web may be omitted in flange area or be allowed its proper value. The first class is used for track-stringers, floor-beams, and spans up to about 35 feet in length, and the second class for longer spans. The longitudinal section of flange plates of uniform strength is a parabola, and to provide for the usual extension of plates beyond the theoretic point, this will be considered to have a length equal to that of the girders between bearing points. Allowing 12½ per cent. loss of area in bottom flange due to rivet-holes, and 3½ pounds as the weight of 1 square inch of iron 1 foot long, we obtain the following:

Weight per lineal foot (w) of flanges:

Longitudinal girder with flanges of uniform section $w = \frac{.8854\, W l^2}{d\, S}$ (5)

Longitudinal girders with flange plates of uniform strength,

$$w = 2.361\, A + \frac{.5903\, W l^2}{d\, S} \quad (6)$$

Cross-girders with flanges of uniform section, $w = \frac{7.083\, W p\, x}{d\, S}$ (7)

Weight per lineal foot (w_1) of girder due to web:

1.—Web omitted in flange area for girders of uniform section or of uniform strength.

As no part of the web is included in flanges, the whole must be added, or $w_1 = 40\, t\, d_1$. (8)

2. Web considered in flange area for girders of uniform section.

As $\frac{1}{3}$ of web is included in flanges, $\frac{2}{3}$ only must be added, or $w_1 = 26.67\, t\, d_1$ (9)

3.—Web considered in flange area for girders with flange plates of uniform strength.

As the formula for flanges considers the angles only of uniform section and the balance of uniform strength, and as the web is of uniform section, $\frac{7}{9} \times \frac{1}{3} = \frac{7}{9}$ of web is provided for in flanges and $\frac{7}{9}$ must be added, or $w_1 = 31.11\, t\, d_1$ (10

Weight per lineal foot (w_2) of girder due to stiffeners. (Omit in floor-beams and track stringers.)

The following formulas provide for extra stiffeners at the inside edge of end bearing plates and double allowance at ends of girders:

1.—Average distance apart, 1.5 × depth of girder,

$$w_2 = \left(\frac{2}{3} + \frac{5\, d_1}{l_1}\right) a \tag{11}$$

2.—Average distance, 1.0 × depth of girder,

$$w_2 = \left(1 + \frac{5\, d_1}{l_1}\right) a \tag{12}$$

3.—Average distance, 0.75 × depth of girder,

$$w_2 = \left(\frac{4}{3} + \frac{5\, d_1}{l_1}\right) a \tag{13}$$

4.—Keystone specifications governed by shearing stress,

$$w_2 = \left(\frac{d_1}{4} + \frac{5\, d_1}{l_1}\right) a \tag{14}$$

Weight per lineal foot (w_3) of main girder due to all iron-work exclusive of flanges, web, and stiffeners.

1.—Deck plate-girders, with cross-ties resting on the top flange of girder. For bottom end and masonry plates, web splice plates, cover plates, anchor bolts, track bolts, rivet-heads, end bracing, cross-section bracing, one system of lateral bracing, and all connections.

$w_3 = \begin{cases} 22.0 \text{ pounds for girders having no flange plates.} \\ 26.5 \text{ pounds for girders having flange plates.} \end{cases}$

Add 6.0 pounds for girders braced top and bottom.

2.—Through plate-girders, with cross-struts, cross-ties resting on the bottom flange of girder. For bottom end and masonry plates, web splice plates, cover plates, anchor bolts, track bolts, rivet-heads, lateral struts, lateral rods, and all connections.

$w_3 = \dfrac{260 \times (N+1)}{l_1} + 20.0$ pounds for girders having no flange plates.

Add 5.0 pounds for girders having flange plates.

3.—Through plate-girders, with iron floor-beams and wood or iron track stringers.

A.—Track Stringers.

For packing bolts, track bolts, spikes, washers, and rivet-heads.

$$w_3 = \begin{cases} 5.0 \text{ pounds for wooden stringers.} \\ 3.5 \text{ pounds for iron stringers.} \end{cases}$$

Add 5.5 pounds if stringers have flange plates.
Add 4.5 pounds if stringers are braced laterally.

B.—End Struts.

(Omit if floor-beams are used at ends.)

For end struts, stiffeners, rivets, and all connections of end struts to main girders and track stringers.

$$w_3 = \begin{cases} \dfrac{990}{l_1} \text{ for wooden stringers.} \\ \dfrac{1\,030}{l_1} \text{ for iron stringers on top of floor-beams,} \\ \dfrac{950}{l_1} \text{ for iron stringers on side of floor-beams.} \end{cases}$$

C.—End Floor-Beams.

(Omit if struts are used at ends.)

For end stiffeners, rivets, and all connections of end floor-beams to main girders and track stringers.

$$w_3 = \begin{cases} \dfrac{560}{l_1} \text{ for wooden stringers.} \\ \dfrac{600}{l_1} \text{ for iron stringers on top of floor-beams.} \\ \dfrac{530}{l_1} \text{ for iron stringers on side of floor-beams.} \end{cases}$$

D.—Intermediate Floor-Beams.

For stiffeners, rivet-heads, and all connections of intermediate floor-beams to main girders and track stringers

$$w_3 = \begin{cases} \dfrac{280 \times (N-1)}{l_1} \text{ for wooden stringers.} \\ \dfrac{295 \times (N-1)}{l_1} \text{ for iron stringers on top of floor-beams.} \\ \dfrac{330 \times (N-1)}{l_1} \text{ for iron stringers on side of floor-beams.} \end{cases}$$

Add 3.0 pounds if floor-beams have flange plates.

E.—Main Girders.

For bottom end and masonry plates, web splice plates, cover plates, anchor bolts, lateral bracing and rivet-heads.

$$w_3 = \begin{cases} 19.0 \text{ pounds for girders having no flange plates.} \\ 24.0 \text{ pounds for girders having flange plates.} \end{cases}$$

The preceding formulas and values are applicable to any specification, not only as to unit stresses, but other conditions of calculation, and will give the weights of material with the correctness of careful estimates. The values of w_1, w_2 and w_3 can be easily tabulated and re-

quire no calculation, and the formulas for the value of (w) are certainly simpler and require less labor in calculation than formula (1), given by Professor DuBois.

Economic Depth of Girders.

The economic depth is governed principally by the material in the web and flanges. The slight effect due to end stiffeners, and in some specifications to intermediate stiffeners, will, for the sake of simplicity, be neglected.

We will consider six general cases, as follows:

1.—Longitudinal girders of uniform section, web omitted in flange area,

$$(w + w_1) = \frac{.8854\ W l^2}{d S} + 40\ t d_1 \qquad (15),\quad d = \sqrt{\frac{.8854\ W l^2}{40\ t S}} \qquad (21)$$

2.—Longitudinal girders of uniform section, web considered in flange area,

$$(w + w_1) = \frac{.8854\ W l^2}{d S} + 26.67\ t d_1 \ (16),\quad d = \sqrt{\frac{.8854\ W l^2}{26.67\ t S}} \qquad (22)$$

3.—Longitudinal girders with flange plates of uniform strength, web omitted in flange area,

$$(w + w_1) = 2.361\ A + \frac{.5903\ W l^2}{d S} + 40\ t d_1 \ (17)\ d = \sqrt{\frac{.5903\ W l^2}{40\ t S}} \qquad (23)$$

4.—Longitudinal girders with flange plates of uniform strength, web considered in flange area,

$$(w + w_1) = 2.361\ A + \frac{.5903\ W l^2}{d S} + 31.11\ t d_1 \ (18)\ d = \sqrt{\frac{.5903\ W l^2}{31.11\ t S}} \qquad (24)$$

5.—Cross-girders of uniform section, web omitted in flange area,

$$(w + w_1) = \frac{7.083\ W p x}{d S} + 40\ t d_1 \ (19),\ d = \sqrt{\frac{7.083\ W p x}{40\ t S}} \qquad (25)$$

6.—Cross-girders of uniform section, web considered in flange area,

$$(w + w_1) = \frac{7.083\ W p x}{d S} + 26.67\ t d_1 \ (20),\quad d = \sqrt{\frac{7.083\ W p x}{26.67\ t S}} \qquad (26)$$

The economic depth (d) is obtained in all cases by making the first differential of $(w + w_1) = 0$.

If $t = \frac{3}{8}$ inch and $S = 9\ 000$, we have following values:

$$(21) = \sqrt{\frac{W l^2}{152\ 500}} \qquad (22) = \sqrt{\frac{W l^2}{101\ 600}} \qquad (23) = \sqrt{\frac{W l^2}{228\ 700}}$$

$$(24) = \sqrt{\frac{W l^2}{177\ 900}} \qquad (25) = \sqrt{\frac{W p x}{19\ 000}} \qquad (26) = \sqrt{\frac{W p x}{12\ 700}}$$

from which it will be seen that the depth varies greatly with the conditions assumed in the calculation. Eor example take a 60-foot span $W = 2\ 310$ pounds per lineal foot, t and S as above.

$$\text{Case 2, } d = \sqrt{\frac{2\ 310 \times (60)^2}{101\ 650}} = 9.0 \text{ feet.}$$

$$\text{Case 3, } d = \sqrt{\frac{2\,310 \times (60)^2}{228\,700}} = 6.0 \text{ feet.}$$

The author's formula (2) therefore, which is based on only one condition of calculation, and that one seldom used, except for short spans, is very misleading.

The formulas 21 to 26 inclusive should also be used with caution, for the lightest girder is not necessarily the cheapest one, which we can best illustrate by an example: Take a 60-foot span, Case 4, data as above. The most economical depth by formula (24) is 6.8 feet. This gives slightly less material in web and flanges than any other depth, but when the cost of workmanship is considered it is found to be much too deep. For depths of about four feet and under, the cost of workmanship may be considered practically independent of depth, but for greater depths the increase in cost for punching, assembling, riveting and handling, is quite considerable. This will vary in different shops, but the average may be taken about as given in the following table, showing the effect of workmanship on the economic depth. It will be seen that although a depth of 6.8 feet is economical in material, a depth of 5.5 feet is most economical all things considered.

Depth in feet.	Material per lineal foot of span.			Increase of cost of work due to depth.	Relative cost per lineal foot of span.
	Weight in pounds.	Rate.	Cost.		
4.0	512.8	2.5	$12.82	....	$12.82
5.0	483.7	2.5	12.09	0.13	12.22
5.5	476.6	2.5	11.91	0.25	12.16
6.0	472.5	2.5	11.81	0.42	12.23
6.5	471.0	2.5	11.78	0.62	12.40
7.0	471.7	2.5	11.79	0.87	12.66

The most economical depth of girder when known cannot always be used, as the depth is very often governed by other considerations more important. Professor DuBois in his example of 63-foot through plate girder uses a track stringer 25.8 inches deep. Such depth is seldom allowed to exceed 18 inches, and it is frequently found necessary to limit the depth to 10 or 12 inches, so practically formulas for economic depth appear to be of very little service.

A careful estimate of the 63-foot through plate-girder span taken as an example by Professor DuBois, using the same loads, impacts and unit stresses, gives results as follows :

Track stringers and track bolts	179.8	pounds.
Floor-beams and connections	186.5	"
Main girders and laterals	574.8	"
Total iron-work per lineal foot of span	941.1	"

The results by the writer's formulas are as follows:

Track stringers and track bolts.	Flanges ..... (5) = 54.2		
	Web ..... (8) = 32.2		
	Balance ..... (w_3) = 3.5		
		$= 89.9 \times 2$	= 179.8
Floor-beams and connections.	Flanges ..... (7) = 59.8		
	Web ..... (8) = 56.6		
		$= \frac{116.4 \times 15 \times 5}{63}$	= 138.5
	End floor-beams (w_3) = $\frac{530}{63} \times 2$		= 16.8
	Int. " (w_3) = $\frac{330 \times 3}{63} \times 2$		= 31.4
			= 186.7
Main girders and laterals.	Flanges ..... (6) = 124.4		
	Web ..... (8) = 115.5		
	Stiffener ..... (13) = 22.9		
	Balance ..... (w_3) = 24.0		
		$= 286.8 \times 2$	= 573.6
	Total iron-work per lineal foot of span		940.1

The results obtained by Professor DuBois are as follows:

Track stringers	165.1 pounds.
Floor-beams	134.9 "
Main girders	615.9 "
Laterals	48.6 "
Total iron-work per lineal foot of span ..	964.5 "

The total weight given by Professor DuBois agrees quite closely with the estimate, and if his lateral bracing had been put in at about twenty pounds per lineal foot, its actual weight, the agreement would have been still closer, but it will be seen by an examination of the weights in detail that this is due to a fortunate balancing of errors, which may not often happen, as follows:

	Actual.	Prof. DuBois.	Difference.
Track stringers	179.8	165.1	−14.7
Floor-beams	186.5	134.9	−51.6
Main girders and laterals ..	574.8	664.5	+89.7
Total	941.1	964.5	+23.4

To make convenient use of the writer's formulas, the values of w_1, w_2 and w_3 should be tabulated, then the only ones requiring calculation are those giving values of w (5, 6 and 7). In these, W, which represents the total weight supported per lineal foot of rail, may, in addition to live load, impact, timber and rail, be taken as follows:

For track stringers	100 pounds.
For floor-beams	200 "

For main girders W may be made to include weight of iron-work, if

this is known with sufficient exactness, or it may be made to include any assumed weight of iron-work, or the weight exclusive of iron-work, as may be preferred, and if the assumed weight varies materially from the actual weight, corrections can be made as follows:

Let W = Load assumed.
Let F = Weight of flanges due to W.
Let D = Difference between assumed and actual weight due to W.
Let C = Correction in weight of iron-work.

Then $\pm (D + C)$ = correction of assumed load, and $\pm (D + C) \frac{F}{W}$ = correction in weight of iron-work, or $C = \pm D \frac{F}{(W - F)}$.

As this correction can be applied so easily, it is a matter of little importance whether the load W be assumed correctly or not.

I regret that I will not be able to comment on formulas (3) and (4) for bridge trusses given by Professor DuBois, as the time and space occupied in the consideration of his formulas (1) and (2) for plate-girders is much greater than contemplated.

WILLIAM M. HUGHES, M. Am. Soc. C. E.—I have tested the formula given by Professor DuBois for obtaining the weight of iron bridges with a result as given below.

These bridges, except the two double-track spans, were calculated to carry a load consisting of two 72-ton consolidation engines, followed by a uniform load of 2 000 pounds per foot, and the double-track spans for a load of 2 400 pounds per foot for each track, including engine load.

For w_1 w_3, and $w_1 + w_2 + w_3$, the values are as follows:

Span	=	50	61	75	84	105	120
w_1	=	1 600	1 550	1 475	1 450	1 430	1 415
w_3	=	16	15	24	26	28	29
$w_1 + w_2 + w_3$	=	1 923	1 886	1 800	1 784	1 765	1 773
Span	=	135	165	184	200	142	150
w_1	=	1 400	1 350	1 325	1 300	2 400	2 400
w_3	=	30	41	43	45	60	60
$w_1 + w_2 + w_3$	=	1 728	1 736	1 717	1 698	3 165	3 157

For w_2 I have used the actual weight of the cross-girders and stringers, which vary somewhat with the same panel length, owing to the fact that more or less was allowed for impact according to the location of the bridge, with reference to the liability of running at a high or low rate of speed. I have taken the floor, track, etc., at 300 pounds per foot, which is slightly in excess for the style of floor used on these bridges.

I have also used for the value of w_3 the actual weights, which are considerably less than the formula would give. In applying the formula for girders I found it to give 1 700 pounds for the weight of a cross girder, whereas the actual weight was 2 200 pounds. In applying this formula it requires entirely too much labor to make it of any practical value. This also seems to me an objection to the formula for obtaining

the weight of the trusses; almost any engineer who makes a practice of figuring bridge work has, or can easily obtain, the weight of structures already built, and can thus assume dead loads sufficiently close that he will scarcely ever be obliged to go over the calculations the second time.

Span.	Kind of truss.	Stringer.	Depth.	Number and length of panels.	Weight, less shoes and rollers.	Weight by formula.	Difference per cent.
			Ft. In.	No. Ft. In.			
50	Pratt.	Iron.	6 3	4 12 6	29 860	28 000	— $6\frac{44}{100}$
61	"	"	8 0	4 15 3	37 894	39 405	+ $3\frac{98}{100}$
75	"	"	21 0	5 15 0	48 557	47 025	— $3\frac{16}{100}$
84	"	"	21 0	6 14 0	59 675	59 470	— $0\frac{34}{100}$
105	"	"	20 0	7 15 0	75 276	80 430	+ $6\frac{85}{100}$
120	"	"	22 6	8 15 0	99 894	106 650	+ $6\frac{76}{100}$
135	"	"	22 6	9 15 0	120 515	133 110	+ $9\frac{50}{100}$
165	Whipple.	Wood.	27 0	11 15 0	161 825	148 830	— $8\frac{02}{100}$
184	"	"	28 0	12 15 4	193 251	192 096	— $0\frac{60}{100}$
200	"	"	28 0	13 15 $4\frac{1}{2}$	236 640	232 800	— $1\frac{62}{100}$
142	Pratt.	Iron.	27 0	7 20 $3\frac{1}{2}$	229 700	229 200	— $0\frac{22}{100}$
150	"	"	28 0	7 21 5	237 930	247 200	+ $3\frac{89}{100}$

The 142-foot span weighed somewhat more than it should, owing to the fact that the same section was used for the floor-beams and stringers as for the 150-foot span.

GEORGE H. PEGRAM, M. Am. Soc. C. E.—I desire first to correct Professor DuBois in his statement that in double-track spans "Mr. Hughes found 51 and 59 per cent. deviation from Mr. Pegram's formula."

The facts are these, that for double-track spans I suggested adding 90 per cent. to the weight given by the formula for single track. Mr. Hughes showed the addition as 51 and 59 per cent., making the deviation 20 and 16 per cent. respectively; quite bad enough, however, if true. In replying to his discussion, I endeavored to show that his conclusions were unreasonable on the face, and that the weights on which they were based showed discrepancies among themselves sufficient to destroy their value as standards. Professor DuBois has doubtless been led into this error through Mr. Hughes' table, in which the additions for double track are put in the column of "differences" for single track.

In commenting upon his formula, based upon a constant limiting length of span, Professor DuBois says:

"Here, then, is a simple formula made off hand which satisfies well the entire range of all Mr. Pegram's examples quite as well, in fact, as his own series of formulas with their carefully determined constants."

Now the only formulas which I presented were for the entire weight of the bridge, while Professor DuBois' formula is for one truss. Comparatively a simple matter and quite a different thing.

The full text of my indorsement of his girder formula is this: "The

results for girders are all that could be asked, * * * but neither the girder nor truss formula gives proper variations for different depths and loads."

Professor DuBois seems to have entirely abandoned the truss formula referred to, and my following remarks will be confined to a discussion of the formula which he substitutes for it, after first calling attention to a serious fault in his proposed live load, which he says "is somewhat in excess of Mr. Pegram's typical consolidation, Class T."

The average weight of his locomotives per foot of track is 3 398 pounds, and of his train, 1 920 pounds. In the "Class T" loading these weights are respectively 3 276 and 3 000 pounds.

While his engine loading is slightly heavier, his train load is very much lighter, in fact it is but little more than half that now running on some roads. The result is that in small spans the difference will be very small, but it will rapidly increase with the span; for example, at 400 feet the equivalent load for Class T will be 800 pounds (about 36 per cent.) heavier. Professor DuBois is quite right with regard to my 32-foot span; the disputed weight should be 352 206 pounds, which agrees almost exactly with his formula weight.

The special merit which Professor DuBois claims for his formula for the weight of one truss is that it is rational and takes into account the panel length, depth of truss and style of truss, and is a reliable guide in deciding upon these elements.

He gives a number of examples to show the accuracy of the formula in giving total weights, which are quite satisfactory as far as they go, but equally good results over the same range could be obtained with a purely empirical formula. There are no examples of different styles of trusses, none showing the effects of different panel lengths, and only one for different depths, the 255½-foot span. The weights of one truss of this span are given for depths of 29 and 38 feet, all other conditions being the same. It should therefore be a good check.

The difference between the formula weights, it will be seen, is 16 671 pounds, and that between the actual weights 10 613 pounds, making the formula 57 per cent. in error.

It will be interesting to check the application of the formula for different panel lengths and for the combined effects of different depths and panel lengths, and to this end is submitted a study of a 255½-foot span with twelve combinations of panel length and depth. The conditions are practically the same in all these spans, only such changes being made in the construction as would occur in building them.

The live load assumed in calculations is two 86-ton Pennsylvania Railroad consolidation engines followed by a train weighing 3 000 pounds per foot, with the usual additions for impact. The stresses are 10 000 pounds in tension and 8 000 pounds in compression, reduced in chords and posts by the usual Bouscaren-Rankine formula. The wind pressure

is taken at 30 pounds per square foot on twice the surface of one truss, together with a moving train surface averaging ten feet high. Wind strains are taken at 50 per cent. greater stress than given above.

Regarding the construction: The trusses are Whipple; the diagonal ties of the 14-panel spans for depths of 29 feet and 32 feet are in one

WEIGHTS OF 255-FOOT 6-INCH SPANS OF DIFFERENT DEPTHS AND PANELS.

DEPTH.	29 feet.	32 feet.	35 feet.	38 feet.
Ten panels of 25.55 feet:				
Top chord	137 698	125 968	116 380	108 186
Bottom chord	90 934	81 548	74 746	68 666
End posts	34 139	34 771	35 979	37 750
End-post struts	3 288	3 376	3 484	3 607
Com. posts and suspenders*	29 684	33 525	37 146	41 178
Diagonal ties	59 681	61 452	60 112	58 734
Pins	10 000	10 000	10 000	10 000
Trusses	365 424	350 640	337 847	328 121
Stringers	75 380	75 380	75 380	75 380
Floor-beams	25 275	25 275	25 275	25 275
Portal and lateral struts	10 201	13 651	14 591	15 476
Lateral rods	10 334	11 416	11 500	11 612
All other iron	12 500	12 500	12 500	12 500
Total span	499 214	488 862	477 093	468 364
Twelve panels of 21.3 feet:				
Top chord	138 664	126 793	117 222	108 673
Bottom chord	90 720	82 199	75 265	69 174
End posts	30 708	31 990	33 233	34 343
End-post struts	3 180	3 294	3 413	3 525
Com. posts and suspenders*	37 776	42 857	47 900	54 740
Diagonal ties	59 656	61 321	60 532	59 831
Pins	12 000	12 000	12 000	12 000
Trusses	372 704	360 454	349 565	342 286
Stringers	62 416	62 416	62 416	62 416
Floor-beams	28 626	28 626	28 626	28 626
Portal and lateral struts	11 867	16 057	16 997	17 882
Lateral rods	11 060	11 870	11 924	12 800
All other iron	12 500	12 500	12 500	12 500
Total span	499 173	491 923	482 028	476 510
Fourteen panels of 18.25 feet:				
Top chord	133 733	122 181	113 154	105 948
Bottom chord	91 307	82 875	73 310	69 220
End posts	28 083	29 820	31 634	32 414
End-post struts	3 090	3 219	3 341	3 452
Com. posts and suspenders*	44 752	51 200	57 218	63 609
Diagonal ties	59 591	59 129	61 572	61 753
Pins	10 560	10 560	13 500	13 500
Trusses	371 116	358 984	353 729	349 890
Stringers	51 748	51 748	51 748	57 748
Floor-beams	31 333	31 333	31 333	31 333
Portal and lateral struts	13 533	18 463	19 333	20 288
Lateral rods	11 986	12 990	13 065	13 234
All other iron	12 500	12 500	12 500	12 500
Total span	492 216	486 018	481 708	478 993

*The hip suspenders are made stiff, similar to posts.

length; those of the other spans are in two lengths, but the pin does not pass through the post. The top struts are lattice girders of the depth of the chord. In the 29-foot depths they are connected with the posts by angle-iron knee braces, in the other depths there are struts between the posts and diagonal rods between these and the top struts. The item "all other iron" is made up of bed-plates, 4 524 pounds; end shoes, 4 256 pounds; rollers, 1 220 pounds; and bolts, washers, loose rivets, etc., 2 500 pounds. A prototype of one of these spans actually built has been used as a check.

Some interesting relations are shown in these weights. It is evident that the panel length has practically no effect upon the total weight. It is also evident that changes in depth affect the weight less as the generally assumed economical angle of inclination of the ties (45 degrees) is approached.

The following table gives the formula and actual weights of one truss of the four extremes of the above table. The first two are those of Professor DuBois' tables, previously referred to. In all cases $(w_1 + w_2 + w_3) = 1\,943$ for one truss.

WEIGHT OF ONE TRUSS.

	Formula.	Actual.
14 panels, 29 feet deep	186 392	185 558
14 " 38 "	169 721	174 945
10 " 29 "	171 619	182 712
10 " 38 "	148 164	164 060

We have seen that with 14 panels, and depths of 29 and 38 feet, the formula is in error 57 per cent. in giving the difference in weight. For a depth of 38 feet, and 10 and 14 panels respectively, it is in error 100 per cent., and for a depth of 29 feet, with 10 and 14 panels, the error is too large to consider. Comparing the 14-panel truss, 29 feet deep, with the 10-panel truss, 38 feet deep, the formula is in error 78 per cent. With such results we are led to inquire whether the formula is really a rational formula.

Professor DuBois has rendered an answer to this quite easy by inviting special criticism upon his equations for the value of the square of the radius of gyration, upon which he says the accuracy of his formula depends.

It will be necessary, I think, to consider but one of these vital equations, viz.:

$$r_1^2 = \frac{(w_1 + w_2 + w_3)\, p\, C}{a^t\, \mu}$$

This equation has no rational basis. It is an assumption in justification of which Professor DuBois appeals to the results of the application of his formula. Such an appeal would be admissible in a purely empirical formula, but certainly not in the vital step of a rational formula. The equation has a definite meaning, however, and can be readily

checked. a^1 is a constant and we can substitute the values of the other factors and see how "constant" it is. I shall take a few spans that have been used in the paper under discussion.

I make C for

$$\text{Pratt truss} \ldots = p^2 (N-2) \left(N + \frac{7}{2} - \frac{6}{N} \right)$$

$$\text{Whipple truss} = p^2 (N-2) \left(N + 5 - \frac{12}{N} \right)$$

The 104-foot and 150-foot spans are Pratt, and the 255½-foot spans, Whipple. In all examples $\mu = 8\,000$, and $r_1{}^2$ is taken from the strain sheet.

The following values of a^1 are obtained in solving the equation:

Span.	N	p	$w_1 + w_2 + w_3$	$r_1{}^2$	a^1
104 feet.....	6	17⅓ feet	2 090	15.5	2 984
150 "	9	16⅔ "	1 950	22.5	4 152
255½ "	14	18¼ "	1 943	52	6 179
255½ "	10	$25\frac{11}{100}$ "	1 943	52	8 600

Such variations in the value of a^1 would seem sufficient to seriously affect the rationalism of the general formula. Its aim is results, and I cannot see that its value is impaired by its empiricism provided proper results can be obtained. It already has a wide application, as Professor DuBois has shown, and I hope it will be modified to give the nice differences which he desires.

A formula which would take into account variations in panel lengths and depths would be very interesting and valuable, but would probably never be used to determine the proportions of bridges. There are too many other considerations than weight entering into the problem. It is fair to assume that the proportions adopted in practice, which are the outgrowth of sharp competition, will always be the best criteria of economy, although they may not be of excellence, and the engineer who has to decide upon these most important elements in the design of a bridge might naturally be expected to know what is being done in practice.

The weight, although the largest factor in the cost of a bridge, cannot be taken as a measure of economy. It is quite probable that the heaviest span in the table of 255½-foot spans (the one with 10 panels, 29 feet deep) would prove the cheapest. It would also be the best, except for the flat inclination of the diagonal ties, to which there seems to be a growing objection.

I can see no "practical and constructive reasons" limiting the panel

to 25 feet, as Professor DuBois suggests. Within the past few years some very excellent bridges have been built with 30-foot panels. In a measure I agree with him in limiting the depth.

Mr. T. C. Clarke, M. Am. Soc. C.E., in his discussion of Mr. Joseph M. Wilson's paper, "On Specifications for Strength of Iron Bridges," in Society Transactions for June, 1886, gives an excellent epitome of "the principal improvements in bridges," from which I quote:

"V. Improvements in design.

"1. Less depth of truss.

"2. The general use of long panels."

These two lines contain the essence of goodness in modern bridges.

HENRY B. SEAMAN, Jun. Am. Soc. C. E.—In the development of his formula, Professor DuBois has calculated his weights upon a theoretical basis, and, it appears, omits the consideration that posts and eye-bars are invariably longer than he has provided for, the posts extending a foot or more longer than assumed, and the eye-bars requiring from 16 inches to 6 feet additional material for the formation of the eyes. Does he not omit to provide for the excess at pin joints altogether?

The percentage of increase due to this I think would be quite appreciable in so accurate a formula.

Professor DuBois' method for finding uniform loads for spans, say 20 feet and under, will not produce "equivalent" loads; they are equivalent in weight, but not in moment.

G. H. THOMSON, M. Am. Soc. C. E.—An old formula is $(fL + C) =$ weight per lineal foot of single-track girder in pounds where

L is the extreme length of girder.

C is a constant.

f is a factor, variable with loading and unit stresses allowed.

C in deck plates covers lateral and vertical sway, including ends, and is uniform for $L = 20$ to 80 feet. In through plates it covers laterals, knees and floor; or it can cover laterals only, and the floor-beams and track-stringers can be numerically added, as the floor is more or less variable in weight, depending upon the specification for loads, and lengths of floor-beams and track-stringers.

For heavy plate work I use $C = 60$ pounds for deck-plate and about three hundred and eighty pounds for through plates and the girders stock, with weight correspondent to the following values of f:

For $L = 25$ feet, $f = 16.0$ decreasing 0.2 per lineal foot for each foot increase of L, making $f = 11.0$ at $L = 50$ feet.

After $L = 50$ feet, f decreases at the rate of 0.1 for each foot increase of L, becoming 9.0 at $L = 70$ feet.

General formulas for weights of truss bridges are misleading, and fail because they are not based on correct principles.

A. J. DuBois, Jun. Am. Soc. C. E.—I regret that Professor Waddell has been unable to test my formula as applied to his examples, and especially that it has not occurred to him to state whether I am correct in asserting that the numerator of the strut formula used by him was taken with a sliding factor, in accord with the expression I have used for it. If this point had been decided favorably, I should be justified in claiming the agreement shown in the comparison of the formula with his results. As it is. I can at present merely call attention to the agreement as more or less corroborative.

The other points alluded to by Professor Waddell are certainly of sufficient importance to warrant discussion, but not in the present connection. It was in the hope that my formulas might furnish a reliable basis for the discussion of some of the important points of bridge design, such as depth, panel length, etc., that I presented them.

The main points to be decided are the reliability, scope and rationality of the formula. I do not present them, as several seem to suppose, as being specially quick of application, although I do not think they will be found so tedious or time-consuming as to render them unavailable. My object was to present formulas which should be, so far as possible, rational in form, and which, if proved to be accurate, might serve as guides in questions of practice. As to ease of application, if the other points are satisfactory, there is no trouble, with the aid of proper tables, in using the formulas at all comparable to the labor which would otherwise be requisite to properly settle the same questions.

The point made by Mr. Gottlieb and others, that the "best depth," properly so called, is that which is cheapest, not necessarily that which is lightest, is too sound and obvious to need comment. But it seems to me that the proper basis for settling upon the one, must involve the determination of the other. Such a process as Mr. Morse has employed, would indicate that the modification of depth due to price is but little for plate girders, and moreover that it is a nearly constant percentage. If such a result could be satisfactorily established, a formula which goes directly the one depth would be a practical solution of the other. I think, however, that Mr. Morse has not satisfactorily taken into account the item of labor, and that such a method as that employed by Mr. Thacher is more to the point, where the increase of cost due to depth is considered. Here again the basis of the solution is a knowledge of the "lightest depth;" and a formula which will give weight with sufficient accuracy for different depths is therefore very desirable.

Let me first discuss this point of accuracy so far as the various comparisons enable me to do so.

Professor Ricketts has given a comparison of results of the formulas for plate-girders. Unfortunately, not enough is given to make the comparison in any sense decisive, or to justify his conclusions. Only the length, depth, shop weights and locomotive system appear to have

been certainly known. The other data required by the formula are, I understand, more or less assumed, and may or may not be in accord with the actual cases. Of course I presume that they are assumed tolerably close, but so are the results also; and the main point is still undecided, as to whether the results are not as close as they ought to be. Professor Ricketts has taken the wind-bracing as given by my formula, not the actual weight. This alone might well make all the difference, as is shown by Mr. Jenning's comparisons. Where the wind-bracing may vary for same span, and within 6 inches of same depth, from 885 pounds to 2 150 pounds, it is essential in testing the formula to know what it actually did weigh. So for the weight of floor Professor Ricketts has taken 400 pounds. This may, of course, be correct, but for deck plate-girders it would seem rather small. If a formula depending on many data is not tested for the actual values of all the data, the results cannot be expected to exactly coincide. Had there been such coincidence, under the circumstances it would be pretty clear evidence not for, but against the formula. As it is, judging from the results of others, I am inclined to believe that the formula results represent very well, not the actual weights, but what would have been the actual weights if all the data assumed had been actually used in the designs.

Mr. H. C. Jennings has given two tables which are of value. In both the weight of bracing, as given by my empirical formula, has been used, instead of the actual weight. In other respects, with perhaps one very important exception, the actual data have been inserted in the formula. If we use the actual weight of bracing, and leave out the bed-plates and bolts, we have the following results:

Span.	Weight of girders.		Total weight.	
	Actual.	Formula.	Actual.	Formula.
41	17 035	15 138	20 540	18 643
49.5	22 476	22 342	26 308	26 174
60	33 576	33 370	38 777	38 571
63	36 694	36 440	42 081	41 827
68	44 160	43 576	51 697	51 113

The agreement here is as close as could be reasonably expected of any formula. The variations, and especially the fact that all the weights of girders alone are less than the actual, would seem to indicate variations in R and a somewhat smaller actual value for R than that used in the formula. The value of R used in the formula was 9 000 pounds, the same value as used by Mr. Jennings. The only differences are using the actual weight of wind-bracing and omitting the bed-plates and bolts. The clear depths are also used in the formula; using the effective depth would give somewhat greater values and make the agreement even closer.

My formula was the clear depth instead of the effective, and disregards the web, on the supposition that in actual designing the web is not disregarded and the effective depth is used, the two errors tending to balance. The web is taken at ½ inch in the formula to allow for splice-plates, stiffeners, etc. As the girders in the preceding table were actually designed disregrading the web, and probably for effective depth, we may expect to find a slight excess over the formula. The formula results are, in fact, a little small. If the girders had been designed taking the web into account, the formula results would be a little large. In either case the formula would give very good results.

In Mr. Jennings' Table No. 2, the girders were figured for the moment of inertia of the net flange section, with average value of R, 10 000 and 9 000 pounds. It will be noticed that the formula results for girder alone, as given by Mr. Jennings, are much too small. I fail to see why a formula which gives correctly the weight of a girder, with constant flange section and ½ inch web, web disregarded in flange strain, should give a less result than the same girder, probably for varying flange section, and certainly not more than ⅜-inch web. It would seem that the test is not sound in some particular. I think the trouble is to be found in the value of R used. It is also to be noted that, through a misconception, the equivalent load for the locomotive system assumed was not used by Mr. Jennings, but only the total load occupying the span. It would seem that if the moment of inertia of the flange section is used with R 10 000 and 9 000, then in the formula we should take for a fair test somewhat more than $\frac{3}{4}$ R, or, say $\frac{5}{6}$ R, or 8 450 and 7 500. Our formula is made out for our method of figuring, and allows for that method an excess for stiffeners, splice-plates, etc. To compare it with another method of figuring it should be tested for equivalent values. It is hard to see how it can possibly give much less than actual weight under any system of design. Taking R at 8 450 and 7 500, and the equivalent loads, and using the actual values of the bracing, and omitting bed-plates and bolts, we obtain the following values:

TABLE No. 2.

Span	Weight of Girders.		Total Weight.	
	Actual.	By formula.	Actual.	By formula.
a 45	20 236	20 248	21 599	21 511
b 30	8 816	9 128	9 701	10 013
c 45	20 664	21 296	23 484	24 116
d 30	11 284	10 034	13 434	12 184

A slight change in the uncertain value for R would, it seems to me, remove such discrepancies as appear.

I do not wish to seem to put more stress upon the accuracy of the formula results than they will bear, but when the results of Mr. Pegram's comparison, given in my paper, are thus apparently confirmed by Mr. Jennings' and Mr. Thacher's, as we shall see presently, and I may add by numbers of cases of my own, I cannot but feel confident that Professor Ricketts' results would show as well if he or I only knew just what data to use. It also appears probable that the accuracy is sufficient to admit of confidence in the conclusions derived from the formula as to "least weight depth," if I may be allowed the expression, to distinguish from least cost or "best depth," provided the formula itself is sufficiently rational.

Mr. Thacher gives for the weight of flanges and web of a longitudinal girder the expression, using his notation,

$$w + w_1 = \frac{a\,W l^2}{d\,S} + b\,t\,d,$$

where a and b are constants, depending upon whether the flanges are of constant cross-section or uniform strength, and whether the web is disregarded or not in figuring the flange area.

He recognizes that in this expression W really contains the dead weight itself, and gives a method of trial and correction by means of which it may be correctly assumed. Would it not be as well to insert it at once in the expression and solve?

Thus, $$w + w_1 = \frac{a\,(W + w + w_1)\,l^2}{d\,S} + b\,t\,d,$$

and hence $$w + w_1 = \frac{a\,W l^2 + b\,t\,d^2\,S}{d\,S - a\,l^2}.$$

This expression is identical with my own. I do not find it necessary to change the constants to fit different methods of designing. This of course is strictly an error. If it is considered proper to do so it can be easily done, but the results of comparisons seem to indicate that for our purpose it is hardly worth while. The errors made in my expression are well summarized by Mr. Thacher himself, and it is hardly necessary to say that I admit them all. But how about their effect? By assuming flanges of constant cross-section, I am correct for short girders and get too much for long ones. By taking the web ½ inch I always get too much; by taking the total length for effective length I get too much; by not allowing for effect of web I get too much; by disregarding rivet-holes I get too little, and so also by taking the depth instead of effective depth. I thus have a surplus amounting to but little for light girders, and increasing, as it should, with span and depth for heavy ones, which goes far to cover connections, splice and cover plates, stiffeners, etc., To claim exact compensation is of course absurd. To claim equal accuracy with special formula designed to cover special cases is equally ab-

surd; but I think I can claim partial compensation, and an accuracy quite satisfactory. Mr. Thacher's comparison well illustrates this.

In his estimate for the 63-foot girder, there is, I think, a slight error in his weight of the flanges of the stringers. Taking my tables, the load is 58 066 pounds, or 3 871 pounds per foot, and adding say 160 for dead weight, we have 4 031 pounds. By Mr. Thacher's formula (5) we then have 49 pounds instead of 54.2.

Also in the web of the floor-beams, 56.6 pounds would correspond to a thickness of $\frac{1}{2}$ inch. Taking the thickness at $\frac{3}{8}$, we should have only 42.45 pounds.

Making these corrections, I would make the comparison thus:

		Thacher.	Formula.
Track stringers and track bolts.	Flanges and web	162.4	165.1
	Balance (tc_3)	= 7.	
Floor-beams and connections.	Flanges and web	121.7	134.9
	Connections, etc	= 48.2	
Main girders and laterals.	Flanges and web	479.8	615.9
	Laterals, etc.,	= 48.0	48.6
	Stiffeners, etc.,	= 45.8	

Now here is evident not a fortunate balance of errors, but an allowance or surplus for stiffness and connections, which, if not exact, at least varies, as it should, with all the dimensions, and gives in this case, as in all the others already examined, a close result. For the track stringers the formula gives a close result. The excess due to web of $\frac{1}{2}$ goes far to balance the allowance for connections, and would go farther if the web were considered in flange area. The same holds true for the floor beams. For the main girder we have also an additional excess due to taking the flanges as of constant cross-section. This excess is of course liable to vary more or less, but it seems to me that after providing for stiffness, splice plates, etc., the small residue can never be a source of great error, and this conclusion seems justified by the results of the comparisons of formula with actual results, and explains the deviations, never very large, which appear.

In his remarks upon "least weight" depth, Mr. Thacher gives the depth for (23) as $d = \sqrt{\frac{.5903\ W l^2}{40\ t S}}$. This seems to me slightly in error. It assumes the term 2.361 A as unaffected by d. But really the constant flange angles support a certain proportion of the whole load, say $a\ W$, and this would give us $d = \sqrt{\frac{.295\ W l^2 (2 + a)}{40\ t S}}$, and this again is not strictly correct, because the dead weight is itself included in W. We should strictly have

$$d = \frac{.295\ (2 + a)\ l^2}{S} + \sqrt{\frac{.295\ W\ l^2\ (2 + a)}{40\ t\ S} + \left[\frac{.295\ (2 + a)\ l^2}{S}\right]^2}$$

where W does not include the dead weight. The effect of both corrections would be to reduce the difference in depth between case (2) and case (3). It seems to me then that I am not without justification in avoiding a change of constants to fit different cases, though, if thought necessary, this can easily be done. With such change, and instead of our formulas for wind-bracing, usual to suit different styles, my formula would become essentially the same as Mr. Thacher's method. Unless actual tests show this to be imperative, I should dislike to do it. All tests thus far given do not, it seems to me, indicate such a necessity.

Whatever may be thought of the accuracy of the formula and method as applied to plate-girders of large span, as far as regards track stringers and cross-girders, the results, it seems, ought to be quite good. If so, then at least the floor system can in any case be accurately estimated, and by the aid of tables similar to those given in my paper, very speedily estimated also.

It remains only to discuss the truss formula. With the values for A once made out and tabulated, this is ready enough of application. Its accuracy seems very thoroughly established if the comparison with Mr. Pegram's cases and the results of Mr. Hughes are considered, so far at least as the scope of these comparisons extend. From the latter we see that in twelve spans of all sizes, of different styles, and built according to different specifications, aggregating 1 521 017 pounds in all, the formula gives an aggregate of 1 524 216 pounds, or only about 3 200 pounds in excess. The fluctuations, almost without exception, are such as can be attributed to fluctuations in μ, or to the excess of actual sections over those required. Even an accurate, perfectly unexceptionable, rational formula must show such variations above and below the actual weights. That the formula should be entirely rational is too much to expect. That it is mainly rational, and in form wholly so, I believe.

In my criticisms of the 104-foot span alluded to by Mr. Jennings, I did not of course imply that the depth chosen was not the best as determined by the special circumstances, but simply used it as an illustration of how the effect of change of depth could be easily studied in connection with change in panel length.

Mr. Pegram is quite correct as to my misunderstanding of the 51 and 59 per cent. deviations. It was an inadvertence on my part. As Mr. Hughes gives these same spans with a deviation by my formula of 0 and $+ 3\frac{1}{4}$ per cent., I should be loth to consider them as "unreasonable on their face." They may indeed not conform to certain standards, but, what seems of more importance, the formula conforms itself to their construction, as a rational formula should do with any span, standard or not, when actual data are inserted.

In the remark quoted by Mr. Pegram as to the "simple formula made off-hand," I fail to see the force of his reply. I had already in my paper given a formula for truss weights, which agreed well with actual weights, and shown that by its aid and my tables total weights were also given with equal accuracy. It seems to follow that if the simple formula alluded to also gives the truss weights closely, precisely the same use of it as of the other would likewise closely give total weights. It is only necessary to add to the truss weight per foot, $w_2 + w_3 - 200$ to get half total weight per foot.

This simple formula is indeed thoroughly rational as far as it goes. It is well known in the history of the subject. For any given system and standard specifications the limiting length may be taken as practically constant, and I am convinced that with a series of values for L to match Mr. Pegram's coefficients it will give truss weights quite accurately, and, combined with any good method of estimating floor, will give with equal accuracy total weights.

In the partial quotation to which he objects, where he indorses the girder formula, it did not occur to me as necessary to quote the remainder, because it referred in part to another truss formula not given in my present paper; and that part of it which referred to the girder did not seem to me, in view of the causes of variation I have already alluded to, entirely justified by the comparison. It is, however, quite in order that Mr. Pegram should call attention to the limitation of his indorsement, and if my statement of it is likely to give rise to misconception, it is unintentional on my part, and I am glad to be corrected.

In the live-load system referred to as somewhat in excess of Mr. Pegram's, I had in mind the locomotive only. The car loads which follow are designedly light. In the tables referred to in this connection in my paper, I found it just as well to take concentrated car loads, and to take them light. By doing so an increase of tabular values is easily figured out to suit any given specifications. In all my comparisons I have taken the loads as given, and therefore it does not seem to me that the fault is very "serious."

I do not understand the statement that there are no examples of different styles of trusses and none showing the effects of different panel lengths. In the table of Mr. Pegram's cases are two styles, single and double intersection. In Mr. Hughes' there are two styles and also different specifications for these styles. I have noticed in my paper the effect of double track and have given a case of Warren girder, and also given several examples of the effect of change in panel length both for constant and varying depth. It is quite true that I have given no actual weights of different styles myself for comparison with the formula, and it is quite proper that I should not. I wish the formula results to be compared with the actual weights furnished by others, not by myself, and if Mr. Pegram or any one else feels the lack of further examples

for comparison, they are invited and desired to furnish them. That is the object of this discussion.

Mr. Pegram in his comparison of the 255.5-foot span with the results of formula, speaks of percentages of error in the differences of weight. The curve represented by my formula, for the depth variable, is an hyperbola. His comparison shows that at different points there is not exact correspondence, and also that at these points the inclination of the tangent to the curve is not the same as for the actual curve. This may well be; it is even to be expected, and yet there can be substantial agreement for all that. If the results of formula are found to vibrate, with but small error, from one side to the other, and yet within a large range of actual comparison give accurate results, it can be assumed that the formula represents the facts. This is the point to be established, and it seems to me that this is the method by which to test it, viz., by actual comparison. I see no object in Mr. Pegram's method of comparison except it be desired to obtain as large percentages of errror as possible. This method might easily be made to seemingly discredit an exceedingly accurate formula. The percentages are large while the actual errors are easily covered by all the attainable fluctuations in design. More tests of just such results with a large number of spans, so that we may see, not the percentage of error in differences in any one particular case, but the errors in actual amounts for a large variety of cases, are just what is needed. Such tests are furnished to some extent by the tables I have given of Mr. Pegram's and Mr. Hughes' cases, and I am desirous to obtain many more.

The results and the construction of the formula lead me to believe that it is mainly rational, and to be depended upon for the effect of changes in the data within a sufficient range, provided the best constants can be determined. It may well be that this I have not done. But thus far I have no data upon which to base better ones, and the results of actual comparison are certainly very good. It will be observed that there are but two empirical constants, 45 and 202. If I wished then to fit the formula to Mr. Pegram's results, I could at most only make it fit any two, and it would have to take its chances with the rest. Yet, out of Mr. Pegram's fourteen cases of five groups, it is almost exactly right in one case in every group through the whole range. The last case of 320-foot span turns out to have been correctly given by the formula. It is under two per cent. in four cases more, and its greatest variation in the rest is less than 5 per cent. In the twelve cases given by Mr. Hughes, it hits almost exactly three cases, and is below 4 per cent. in all but five cases. Such variations as there are might be owing to slight variations in loading and unit stress, even in an exact formula, and especially to excess of actual sections over those required. This is certainly a good showing, as the cases cover a large range, include different styles, and are built under different specifications.

Mr. Pegram is quite right in giving special attention to the expression for r_1^2, the square of the radius of gyration for the upper chords. I especially invited discussion on this point, and considering its importance, I could have wished that Mr. Pegram's treatment had been somewhat more thorough. When he speaks of it as a mere assumption and having no rational basis, I hardly know how to interpret the words. Does he mean that it is pure assumption to say that r_1^2 increases with the weight of the chord or with the loading, and that such assumptions are irrational? It would seem that the basis is as rational as can be desired. The form only is to be criticised.

Mr. Pegram proceeds to test this expression. To do this he shows that putting for r_1^2 its actual value, the constant in the expression is not constant. Now, it may appear paradoxical to state that if the value of r_1^2 as given by the invention of a constant for a^1 had checked exactly the true values of r_1^2, that the fact would have proved the expression to be incorrect. But such is the case. The value of r_1^2 given by the expression in question is not the actual value for the chords as found from the strain sheet, nor is it meant to be. If it were, then it would itself need to be reduced by a varying amount to allow for details, lattice bars, pin-plates, etc. Mr. Morse has very properly noticed this point, and if the expression as it stands had really stood Mr. Pegram's test and closely agreed with his values, it would have shown a clear case of "overloaded constant," no provision being made for details, and Mr. Morse's objection would have been sustained.

Now, it seems to me no pure assumption, but to have a sound rational basis, to assert that the actual value of r_1^2, as found from the strain sheet, ought to increase with $(w_1 + w_2 + w_3)\ C$, which is proportional to the area, and ought to vary inversely as μ and N. We have then for the strain sheet values,

$$r_1^2 = \frac{(w_1 + w_2 + w_3)\ C}{a\ \mu\ N.}$$

If we test this as Mr. Pegram has done, to see how constant a is, we have the following results.

Span.	N.	p.	$w_1 + w_2 + w_3$.	r_1^2.	a.
104	6	17½	2 090	15.5	28.7
150	9	16⅔	1 950	22.5	29.6
255½	14	18¼	1 943	52	24 2
255½	10	25.55	1 943	52	33.5

A value of a not far from 30 would it seems give a good average value for r_1^2. It is not stated whether the values of r_1^2 given are the

greatest values or average for the span. The variation of a is slight and it can be practically taken as constant. Perhaps if average values of r_1^2 were used in testing, the variations of a would be still less. As it stands, it seems to me that the expression just given for r_1^2 will give very good values for r_1^2, if a is taken about thirty.

Now, as Mr. Morse points out, the details are a varying percentage. If we multiply numerator and denominator by p, we have

$$r_1^2 = \frac{(w_1 + w_2 + w_3)\, p\, C}{a\, \mu\, l}$$

If we take as our varying percentage $\frac{l}{a^1}$, which gives us just such an allowance as Mr. Morse insists on, we have

$$r_1^2 = \frac{(w_1 + w_2 + w_3)\, p\, C}{a^1\, \mu}$$

which is precisely the expression given in my paper. It will now of course no longer check the actual values of r_1^2, and just because of this it does take account of details, and it seems to me the constant is not "overloaded."

As with this single exception everything in the formula is strictly deduced, and the form is thus rational, it would seem that if this single exception is in accord with reason and meets Mr. Morse's objection, and at the same time Mr. Pegram's, an appeal to the results of the application of the formula is quite in order. All such appeals thus far have been quite satisfactory. If the error in differences noticed by Mr. Pegram is considered important, I should expect it to be easily remedied by a better choice of constants, thus bringing the formula curve into close accord with the actual. The main point is whether the form and character of the formula curve is correct. If so, its course can be easily guided. More studies like Mr. Pegram's, of the same span with different depths are needed. If the formula can be fitted to these and still give as good results as at present, it would certainly be a reliable guide in questions of proportion.

The point made by Mr. Seaman, that the theoretical weight does not take account of full length of posts or of additional length of eye-bars required for heads, is well taken. Two ways of meeting it are open. Either to insert the addition as a percentage of increase to the theoretical value for A, or to take account of it in the expression

$$\frac{a\, p^2 + b\, d^2}{(w_1 + w_2 + w_3)\, p}$$

I find that this expression, with the constants 45 and 202 as given, seems to cover both the material required for long struts and this also. The same remarks which cover the objections of Mr. Pegram and Mr. Morse as to value of r_1^2 and "overloaded constant" apply here. The ex-

pression is of the same form as the theoretical value of A, and the results seems to establish that a and b are practically constants.

As to Mr. Seaman's remark upon my method of finding uniform load, I do not catch the force of it. I should certainly in finding "equivalent load," find that uniform load which gives the greatest moment caused by the actual loading. So far as the live-load system is concerned, I do this. To it I add the loading due to floor system. For a span of 20 feet, or any other span, I have intended to give for the uniform loading the equivalent load for the load system adopted, increased for impact, and add to it the floor system. In all my comparisons I have of course taken the load as given by those who furnish the example.

The statement of Mr. Thomson, that general formulas for the weight of truss bridges are misleading, and fail because they are not based on correct principles, seems to me to be itself misleading by reason of its generality. He would hardly assert that the weight of any special span, with given data and specifications, cannot be correctly estimated. But in the derivation of my formula I have followed very closely, step for step, the method which would be pursued in any special case. The principles which hold for the one are applied in the other. The strain in every member is found, its area and volume and weight determined, and then the weights added. Of course it does not follow that a problem which admits of special solution in any given case can always be solved generally, but if not, a good reason can usually be given. Certainly, incorrect principles in the present case cannot be put forward consistently as a reason by any one who uses identical principles in the solution of special problems. My formula is based throughout upon accepted statical principles, and with the single exception of extra material for stiffening, etc., is accurately rational. The allowance for this extra material seems to me also rational in its form, and the results seem thus far to justify it.

I give on opposite page a tabular statement of results obtained by using my formula and tables, which may be of interest. All the spans are double intersection. The "least weight depth" is given in each case.

Span.	No. of panels.	w_1	w_2	w_3	Depth.	Total weight of iron.
100	4	1 625	376	26	25	76 500
	5	1 625	357	33	22.5	78 000
	6	1 625	347	39	20	80 400
	7	1 625	341	46	17.5	84 200
110	5	1 643	365	31	25.5	90 860
	6	1 643	352	38	23	92 180
	7	1 643	344	44	21.5	95 580
120	5	1 637	373	30	28.5	95 500
	6	1 637	357	36	26	96 720
	7	1 637	348	42	24	97 920
130	5	1 625	380	38	32	117 000
	6	1 625	364	41	29	124 280
	7	1 625	353	43	26.5	128 180
140	6	1 610	370	37	31.5	134 400
	7	1 610	357	43	29	139 160
	8	1 610	350	50	27	141 680
150	6	1 588	376	38	33	150 000
	7	1 588	363	44	32	152 100
	8	1 588	353	50	30	157 200
160	7	1 560	368	45	34.5	168 960
	8	1 560	357	51	32	173 440
	9	1 560	350	57	30	178 560
170	7	1 538	374	45	37	186 660
	8	1 538	362	51	35	191 420
	9	1 538	354	58	32.5	197 880
180	8	1 511	367	52	37	208 080
	9	1 511	357	59	35	216 360
	10	1 511	351	65	33	222 120
190	8	1 486	372	52	39.5	228 000
	9	1 486	361	59	37	234 460
	11	1 486	354	66	35	242 440
200	8	1 460	376	53	42	246 800
	9	1 460	362	59	40	252 400
	10	1 460	357	66	38	261 600
210	8	1 435	381	53	44.5	267 540
	9	1 435	370	60	42	275 200
	10	1 435	361	66	39.5	283 080
230	9	1 386	378	60	47	317 400
	10	1 386	369	67	44.5	329 360
	12	1 396	354	80	40	349 600
250	10	1 348	376	68	49	377 500
	12	1 348	360	81	44.5	400 500
	14	1 348	351	95	40.5	429 000
270	10	1 310	384	68	50	428 760
	12	1 310	367	82	49	456 300
	14	1 310	355	96	44.5	487 620
300	12	1 260	376	83	50	549 000
	14	1 260	363	97	50	588 000
	16	1 260	353	110	47	625 800

As to Mr. Thomson's formula for plate girders, it takes the form for total weight $(21 - 0.2\,l)\,l^2 + b\,l$ for spans under 50, and $(16 - 0.1\,l)\,l^2 + b\,l$ for spans between 50 and 80, where $b = 60$ for deck and 380 for through girders.

Mr. Thomson speaks of this as if the first term represented the girder proper and the last term the sway bracing, etc. The formula has no such meaning. It is simply an application of the familiar expression

$$a + b\,l + c\,l^2 + d\,l^3 +, \text{ etc.}$$

which has been made the basis of so many empirical formulas in the sciences. If the quantities to be represented form a straight line, the first two terms are used. If a curve, three or more. Mr. Thomson's formula is but the application of this, and as it has practically six constants for the small range from 20 to 80 feet, it can be fitted accurately in six places to any series of values, and will of course give good intermediate values. The constants thus determined, however, are good only for girders of a certain style and for personal practice, and worthless for any one else.

Thus, take the deck-plate girders given by Mr. Pegram in the February Transactions, 1886, for Class C. If we fit Mr. Thomson's formula to these cases at 20, 50 and 80 feet we have for the weight $\left(7.4 + \frac{l}{290}\right) l^2 + 192\,l$. This formula is very different in the constants from Mr. Thomson's, but gives exceedingly close results for Mr. Pegram's cases, without any change for spans over 50. I presume it would not fit Mr. Thomson's practice, however, while on the other hand Mr. Thomson's formula does not at all fit Mr. Pegram's practice, as will be seen from the following tabulation:

Span.	Formula.	Thomson.	Actual weight.
20	6 827	8 000	6 841
30	12 510	15 300	12 732
40	19 740	23 200	19 283
50	28 530	30 500	28 561
60	38 880	39 600	38 590
70	50 876	48 300	49 983
80	64 448	56 000	64 561

If the problem be merely to reproduce the results of special practice by an empirical formula, this is easily done, and Mr. Thomson's form is a very old and favorite one for all such purposes. It can be fitted to Mr. Pegram's results with much better accuracy than the form he has adopted, as I have satisfied myself by actual trial and comparison through the whole range. The result of any such formula is at best but a tolerable estimate, varying more or less from the actual according as

the case in hand departs more or less in dimensions, loading, etc., from the practice upon which the formula is based.

I believe that a better empirical formula for trusses than any thus far proposed can be given by the form already alluded to, viz.: $w_4 = \frac{w_1 + w_2 + w_3}{\frac{L}{l} - 1}$, where the limiting span L may be given by a simple empirical formula. Such a formula would have the advantage of a rational form, and might therefore be expected to follow better the actual curve. It would also take account of loading and depth as well as span. Even if we take L as constant, the result as we have seen is very good.

I would therefore propose for $\frac{L}{l}$ the form $\frac{\mu\, d}{a\, l^2 + b\, d^2}$, as coinciding with the theoretic form already deduced. Determining the values of a and b from spans 104 and 320 as given by Mr. Pegram, we have for $\mu = 8\,000$, $a = 0.635$ and $b = 25$. Here then is an empiric formula of wider scope, which follows change of depth and loading with considerable accuracy, as shown by the following tabulation. Better values for the constants a and b may undoubtedly be found. The preceding are hastily formed from two extreme cases only.

Span.	Depth.	No. of panels.	$w_1 + w_2 + w_3$	Formula weight of one truss.	Actual weight.	Difference.	Per cent. of difference.
104	24	6	1 820	23 649	23 658	— 9	0
			2 000	26 000	24 458	+ 1 542	+ 6 3/10
			2 090	27 170	25 074	+ 2 096	+ 8 36/100
150	25	9	1 675	44 250	46 032	— 1 782	— 3 87/100
			1 844	48 750	49 628	— 878	— 1 77/100
			1 950	51 600	52 627	— 1 027	— 1 95/100
201.5	28	12	1 565	80 197	85 131	— 4 934	— 5 8/10
			1 776	90 675	93 626	— 2 951	— 3 15/100
			1 946	99 742	100 283	— 541	— 0 54/100
320	34	20	1 605	271 680	273 671	— 1 991	— 0 73/100
			1 798	304 320	299 718	+ 4 602	+ 1 53/100
			2 088	353 280	352 206	+ 1 074	+ 0 30/100
255.5	29	14	1 943	183 193	185 558	— 2 365	— 1 27/100
255.5	38	14	1 943	170 163	174 945	— 4 782	— 2 73/100
255.5	32	14	1 943	176 550	179 616	— 3 066	— 1 7/10

There are undoubtedly other elements than that of weight alone entering into the problem of proportions. But so long as the weight is the basis of all discussion, the effect upon weight of change of dimensions must be of importance. The outgrowth of practice under sharp competition is to be relied upon, but such a formula ought to aid

such growth and give valuable hints to practice. The conclusion of modern practice as to long panels, for instance, is justified by the formula, and such a formula in the beginning might well have facilitated, and even hastened, the conclusion.

If I cannot then altogether agree that the present attempt has been as thoroughly tested and discussed as it should be, or that its value has been thus far conclusively settled, it is not because I am willfully hard to be convinced. Nor would I be thought to treat with scant ceremony the conclusions of those whose opinions are justly entitled to weight as those of acknowledged experts. I trust it will not be considered obduracy if I still ask for further tests.

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DISCUSSION ON FORMULAS FOR THE WEIGHTS OF BRIDGES.*

WILLIAM H. BURR, M. Am. Soc. C. E.—In this paper Professor DuBois has undoubtedly framed a formula of very considerable accuracy, provided the quantities which enter into it and which depend upon other formulas are accurately computed. A formula which gives the weight of iron in any railway truss bridge must have its value based upon two considerations, one of which is its accuracy and the other its ease of application; such a formula would have value in either one of two uses.

In the first place, if it were sufficiently accurate to be used as a basis of a tender, any reasonable amount of complication would not be of importance. On the other hand, if the formula were not sufficiently accurate to give a reliable weight on which the contract price of a bridge is to be founded, any complication in its form or application would render it of little value for approximate purposes.

I believe that there are few engineers who would be willing to hazard an important tender on any formula which has yet been devised, including the one under consideration. The great variety in the specifications now used by railroad companies and their engineers, leads to an equally great variety in the weights of the iron for a span of a given length, whether of a single or a double-track structure.

If it can be shown that the formula of Professor DuBois will give the

* This is a continuation of the Discussion on the Paper on Formulas for the Weights of Bridges, by A. J. DuBois, Vol. XVI, page 191, May, 1887.

weights of iron in spans of different lengths and for different specifications within two or three per cent., it may be considered as sufficiently accurate for use in making tenders in close competition, but not otherwise. It is safe to say that neither this nor any other formula which is or can be devised with reasonable simplicity will fulfill this condition. But if a formula is not sufficiently accurate for the purposes of close estimate, it still may possess considerable value in making those approximate estimates of cost which are so frequently required in ordinary practice. Such a formula, however, should be very simple, and that under consideration does not meet this requirement.

In the first place, we find that in order to obtain a result from the application of Equation 3 in Professor DuBois' paper, at least four quantities which enter it must be computed by other formulas; and we find, furthermore, that one of them, w_1, involves a very considerable amount of labor, and after it is obtained it is entirely an imaginary quantity, and one which possesses no real existence.

After having found these four quantities therefore, with considerable labor, from other formulas, they are to be inserted in a formula of considerable complication, and one from which a numerical result cannot very readily be obtained. It does not appear therefore that the formula can be used for purposes of an accurate estimate, nor yet be quickly and simply used for the purposes of an approximate estimate. I believe it is true that the claim of Professor DuBois for increased accuracy over other formulas is well founded. It is probably by far the most nearly accurate formula which has yet been devised. I only wish to point out the facts that it is not sufficiently accurate to form the basis of a tender in ordinarily close competition, nor is it sufficiently simple to admit of quick use for the purposes of an approximate cost.

The use of an equivalent uniform moving load is a matter of which much has lately been said, and it would seem to appear that it is supposed that there is a real uniform load per foot, which is in some sense or other equivalent to any given system of concentrated moving loads; but such is not the case. There is no such thing whatever as a uniform moving load which is equivalent to a given system of concentrated loads for any length of span, long or short. It is very true that there may be found a uniform load per lineal foot which will produce the same bending moment at any section of a truss as a given system of concentrated loads, or which will give the same shear as that system of moving loads in any given web member; but that so-called equivalent moving load will not give the same moment, or the same shear, in any other parts of the structure.

Nothing can militate against the accuracy of any formula, therefore, with greater certainty, than this assumption of the use of an equivalent moving load, especially if, as in the present paper, the definition of that equivalent load is omitted.

A formula for the weight of iron in a given truss, and under a given system of moving loads, of sufficient accuracy to take the place of a detailed estimate, is very much like an accurate long-column formula; both are absolutely impossible.

A. J. DuBois, Jun. Am. Soc. C. E.—Professor Burr justly remarks that variety in specifications leads to equal variety in the weight of a given span; but he does not recognize that my formula accommodates itself to different specifications and thus actually gives the weight in accordance with the specifications adopted.

If it can be shown that the formula gives weights within two or three per cent., it may be used in making tenders. This condition Professor Burr thinks it safe to say neither this nor any other formula can ever fulfill with reasonable simplicity. Why? The problem is of easy solution in any special case. It rests mainly upon static calculation, which can be perfectly generalized. Why should not the generalized calculation be as reasonably simple as the special solution? Of course, as I have already admitted, it does not always follow that because a special problem can be solved, therefore the problem can be generalized; but it generally does, and when it does not the reasons why can always be clearly stated. If such valid reasons exist in this case they have not so far been clearly stated in this discussion. Indeed Professor Burr's admission that this is "by far the most nearly accurate formula which has yet been devised," would seem to indicate that these reasons are not insuperable. The accuracy is simply the result of following, step by step, in the generalization, the successive operations of the special solution. That the attempt to do this even imperfectly results in the "most nearly accurate formula yet devised," seems to me significant. What *prima facie* valid reasons are there why this may not be still more perfectly done and a still more accurate formula result? With a margin of three per cent. to cover minor irregularities, I am of opinion, after considerable studying of the subject, that this can be done.

Professor Burr over-estimates the labor of using the formula. The four quantities he alludes to can all be tabulated in accordance with any specification, once for all. Any one using the formula in practice would provide first his tables. Any bridge engineer has the data for such tables at hand. These tables once formed, the application of the formula is about as simple and easy as any of the far less accurate and less rational ones now in use. The same objection might as reasonably be made to one proposing logarithms as an aid in calculation. The logarithm must first be computed at considerable labor; but how many compute the logarithms they use? Properly fitted for use therefore, the formula will admit of quick use for approximate purposes, and it is admitted to have greater accuracy. In addition it fits any specifications and takes account of style of truss. It appears to me therefore that it

is worth the trouble to any engineer to furnish the tables which thus render it available.

The error due to equivalent uniform load is committed in the interests of simplicity. Its effect in militating against the accuracy of the formula can only be determined by trial. Such comparisons as have thus far been made certainly do not show that the effect is great.

I believe that the Professor is too scientific to pronounce any problem "absolutely impossible" unless there are valid reasons in the nature of the problem itself which justify the assertion; the problem of perpetual motion, for instance, or squaring the circle. If I am laboring upon such a problem I hope, for the sake of our common humanity, he will spare enough time either privately or publicly to enable me, if not unfortunately too far gone already, to get out of the toils.

To compete with a detailed estimate by any method less complete and thorough is, I admit, "impossible" in the nature of things. But with a margin of three per cent., and a similar method at bottom in both cases, it would seem to me that the question is not closed.

Professor Burr neglects to mention the aid which a rational formula as accurate as this ought to be in determining the influence of variation in dimensions, etc. This, it seems to me, should not be entirely ignored.